Modeling Neural Spiking with Point Processes Nonparametrically: A Convex Optimization Approach

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Characterizing neural spiking activity as a function of environmental stimuli, and intrinsic effects such as a neuron's own spiking history and concurrent ensemble activity is important in neuroscience. Such a characterization is complex and there is increasing need for a broad class of models to capture such details. Point process models have been shown to be very useful in characterizing neural spiking activity [1]. The likelihood of a point process $\{N(t)\}_{t=0}^{T}$ is completely defined by its conditional intensity function

$$\lambda(t|x_t) \triangleq \lim_{\Delta \to 0} \frac{P\left(N(t+\Delta) - N(t) = 1 | x_t\right)}{\Delta}$$

where x_t corresponds to previous spiking activity, $\{N(\tau)\}_{\tau=0}^{t}$, as well as any latent environmental stimuli. Most point process models are parametric as they are often efficiently computable, the parameters may be related back to physiological and/or environmental factors, and they have nice asymptotic properties when $\lambda(t|x_t)$ lies in the assumed parametric class [1]. However, if $\lambda(t|x_t)$ does not lie in the assumed class, misleading inferences can arise. Nonparametric methods are attractive due to fewer assumptions, but very few efficient methods for estimating $\lambda(t|x_t)$ are known. We propose a computationally efficient method for **nonparametric maximum likelihood estimation** when $\lambda(t|x_t)$ is assumed to be Lipschitz continuous [2].

We are given neural spiking activity observations $\{N_i\}_{i=1}^{M}$ once every $\Delta = 1$ ms that result from $\{x_i\}_{i=1}^{M}$, known stimuli and the neuron's own spiking history. $\hat{\lambda}_i$ is the estimate of $\lambda(t|x_t)$ at millisecond i. We minimize the negative log likelihood of the point process subject to the Lipschitz continuity constraints:

$$\min_{\hat{\lambda}} \quad \sum_{i=1}^{M} -N_i \log(\hat{\lambda}_i) + \hat{\lambda}_i \Delta \tag{1a}$$

$$s.t. \quad \left|\log(\hat{\lambda}_i) - \log(\hat{\lambda}_j)\right| \leq K \left\| X_i - X_j \right\|_\infty, \quad i < j, \;\; j = 1, \ldots, M \tag{1b}$$

We show that (1a) is convex, as are the constraints in (1b); so (1) is a **convex optimization problem** and thus **efficiently solvable**. We develop an equivalent problem with **separable** [3, Sec. 3] structure in (1a) and linear structure in (1b) to represent its **dual** in closed form. Thus (1) can be solved using **very efficient unconstrained methods**, such as gradient descent. We apply our method to goldfish retinal ganglion neural data and compare results to inhomogeneous Poisson and inverse Gaussian parametric models. We assess goodness-of-fit via the time-rescaling theorem and measure model uncertainty via bootstrapping [4].

References

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