MULTIOBJECTIVE OPTIMIZATION OF LOW-ENERGY TRAJECTORIES USING OPTIMAL CONTROL ON DYNAMICAL CHANNELS (PART I)

Thomas M. Coffee,* Rodney L. Anderson,† Martin W. Lo‡

We introduce an automated approach to design efficient low-energy trajectories by extracting initial solutions from dynamical channels formed by invariant manifolds, and improving these solutions through variational optimal control. We consider trajectories connecting two unstable periodic orbits in the circular restricted 3-body problem (CR3BP). Using the example of an Earth-Moon libration orbit transfer, this paper demonstrates an algorithm for extracting a range of efficient solutions from dynamical channels using adaptive numerical methods, and describes how the algorithm may be extended using primer vector theory to iteratively improve these solutions into an approximation of the multiobjective Pareto front.

INTRODUCTION

Orbit transfers accomplished using low-energy trajectories frequently exhibit significant trade-offs between the required propulsive impulse (ΔV) and required time of flight (TOF). Mission designers can benefit greatly from knowledge of a range of efficient (non-dominated) options in the multiobjective minimization of ΔV and TOF. The highly nonlinear dynamics that enable low-energy trajectories in multi-body gravitational models also pose challenges to identifying such a range of efficient options. The approach described here generates candidate solutions from dynamical channels formed by intersections of invariant manifolds of the initial and target orbits in the circular restricted 3-body problem (CR3BP), and locally improves these solutions through variational optimal control methods to construct an approximation to the Pareto front of efficient options.

Numerous studies have demonstrated the utility of dynamical channels as a tool for finding efficient trajectories in the unstable nonlinear dynamical regions of the CR3BP. In particular, certain pairs of libration orbits exhibit transfer trajectories with no deterministic maneuvers, and many pairs of mean motion resonance orbits show evidence of transfer trajectories requiring only small propulsive maneuvers. Moreover, examinations of multiple efficient trajectories found by extensive search and numerical optimization indicate a close correspondence with nearby dynamical channels. In the case of transfers between unstable periodic orbits, we interpret these results to mean that such transfers are often geometrically well-approximated by solutions consisting of two ballistic segments joined by an impulse. This motivates the first stage of our approach, in which a global search for low-energy transfers is begun with a global search for single-impulse transfers.

Several methods for trajectory design have employed invariant manifolds to find efficient solutions. Early work explored transfers to libration point orbits using their stable manifolds, and applied Floquet theory to optimize transfers between halo orbits of different energies in the same family. For trajectories to libration point orbits, research has explored construction and optimization of transfers targeting stable manifolds, using differential correction and constrained gradient optimization, thrust-limited tangent steering laws, and thrust-limited optimization by both direct and indirect methods. To transfer between arbitrary unstable

*Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, 77 Massachusetts Ave., Cambridge, MA 02139
†Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Dr., Pasadena, CA 91109
‡Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Dr., Pasadena, CA 91109
periodic orbits connected by dynamical channels, several studies have used Poincaré sections to identify desirable channels, and others have applied direct optimization methods to refine such solutions. Other investigations for particular mission types have parameterized limited families of impulsive dynamical-channel transfers with a small set of variables subject to numerical optimization. Another recent approach constructs transfers between invariant manifold segments selected according to a two-body heuristic criterion, which can be likewise optimized according to a small number of parameters.

Our method leverages dynamical channels to explore the space of efficient low-energy solutions across a wide range of the objective parameters. We introduce a global optimization procedure based on adaptive numerical continuation to search for intersecting segments of invariant manifolds propagated up to a desired TOF bound. In examples of interest, the unstable and stable manifolds of initial and target orbits can form multiple disconnected families of dynamical channels that offer different trade-offs between \( \Delta V \) and TOF. Extracting initial solutions representing multiple such families yields a dynamically motivated sample of trajectories distributed across a region expected to lie near the \( \Delta V \)-TOF Pareto front. The local multiobjective optimization of these trajectories then yields an approximation to the Pareto front within the desired bounds.

We describe a method for local optimization based on the indirect primer vector approach to optimal control, originally developed by Lawden for two-body trajectory optimization. Prior investigations have applied this approach to both impulsive and thrust-limited trajectories with three-body dynamics. We begin by adapting the general strategy developed in the Apollo-era research of Lion & Handelsman and Jezewski & Rozendaal to construct locally optimal impulsive transfers in the two-body problem. This iterative procedure employs the time profile of the primer vector on a suboptimal trajectory to guide the differential adjustment or addition of impulsive maneuvers to satisfy the necessary optimality conditions. In a subsequent paper, we will describe and demonstrate a multiobjective form of this method that can be used to improve the solutions extracted from dynamical channels into an approximation of the Pareto front.

In this paper, we demonstrate the first stage of our approach applied to extract dynamical channel solutions for a libration point orbit transfer in the Earth-Moon system. For clarity of illustration, we consider a planar transfer between Lyapunov orbits about the \( L_1 \) and \( L_2 \) libration points, though the method is not limited to planar transfers. These two orbits have differing values of Jacobi’s integral, so that no propulsion-free transfer exists between them in the CR3BP. However, our method discovers a range of impulsive transfer solutions that illustrate the trade-off between \( \Delta V \) and TOF over a range of transfer geometries.

**METHODS**

Our approach breaks down the search for efficient transfer trajectories into two stages: First, we construct a set of initial solutions representing families of dynamical channels connecting the initial and target orbits. These solutions are found through a global optimization approach based on adaptive numerical continuation. Second, we improve these solutions by an iterative process based on local optimality conditions formulated in terms of primer vector theory. The resulting locally optimized solutions are taken to approximate a portion of the globally efficient solution set.

**Constructing Transfers from Dynamical Channels**

The circular restricted 3-body problem (CR3BP) models the mutual gravitation of three bodies as point masses, where the third body has negligible mass, and the primary and secondary bodies move in a circular orbit about their gravitational barycenter. In applications, the third body typically represents a spacecraft or small object, while the primary and secondary bodies typically represent a star-planet or planet-moon pair. We choose a system of units and synodic rotating coordinates \( x, y, z \) so that the secondary body has mass \( \mu \) and the primary has mass \( 1 - \mu \); the primary is fixed at coordinates \((-\mu, 0)\) and the secondary at \((1 - \mu, 0)\); and the equations of the third body’s motion are given by
\[
\begin{align*}
\ddot{x} + 2\dot{y} &= \partial_x \Omega(x, y, z) \\
\dot{y} - 2\dot{x} &= \partial_y \Omega(x, y, z) \\
\ddot{z} &= \partial_z \Omega(x, y, z),
\end{align*}
\]

where \(\partial_x \Omega, \partial_y \Omega, \partial_z \Omega\) denote the derivatives with respect to \(x, y, z\) of
\[
\Omega(x, y, z) = \frac{x^2 + y^2}{2} + \frac{1 - \mu}{r_1} + \frac{\mu}{r_2} + \frac{\mu(1 - \mu)}{2},
\]

with distances from the primary and secondary massive bodies to the third body given by
\[
\begin{align*}
r_1 &= \sqrt{(x + \mu)^2 + y^2 + z^2} \\
r_2 &= \sqrt{(x - 1 + \mu)^2 + y^2 + z^2},
\end{align*}
\]

respectively. The planar CR3BP (PCR3BP) is the restriction of the CR3BP to the plane \(z = 0\).

In the above coordinates, the CR3BP has a first integral (conserved quantity) traditionally expressed as the Jacobi integral
\[
C = 2\Omega(x, y, z) - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2).
\]

The existence of the Jacobi integral implies that any trajectory of the third body lies on a 5-dimensional manifold imbedded in 6-dimensional phase space. In the planar problem, each trajectory lies on a 3-dimensional manifold imbedded in 4-dimensional phase space. Any transfer between orbits with different values of Jacobi’s integral requires some propulsive impulse.

Unstable periodic orbits in the CR3BP have 2-dimensional invariant manifolds constituting those trajectories that asymptotically converge to (stable manifold) and diverge from (unstable manifold) the periodic orbit. The invariant manifolds of a periodic orbit represent all states reachable from the periodic orbit moving backward and forward (respectively) in time with no propulsive maneuvers. When the stable manifold of one periodic orbit intersects the unstable manifold of another periodic orbit, we term this connection a dynamical channel from the second orbit to the first. If the two manifolds intersect in position space only, a trajectory from one orbit to the another may be accomplished by a single propulsive maneuver at the intersection point, cancelling the difference in velocity. If the two manifolds intersect in the full state space, there exists a corresponding trajectory with no propulsive maneuvers.

**Generating Solutions by Numerical Continuation**

We construct candidate transfer solutions by searching for dynamical channels formed by the stable manifold of the target orbit and the unstable manifold of the initial orbit. Describing each invariant manifold by two free parameters, this search may be initially formulated as a four-dimensional global minimization of the position-space distance between points on the manifolds (for smoothness, we use the squared Euclidean norm). The highly nonlinear geometry of the invariant manifolds motivates a specialized approach to this global optimization based on adaptive numerical continuation.

Invariant manifolds of a periodic orbit in the CR3BP have no effective global representation, either explicit or implicit: rather, they are locally defined in relation to the orbit by the equations of motion. The manifolds of an orbit can be approximated near the orbit by the linearized dynamics around the orbit, expressed by the monodromy matrix. The eigendecomposition of this matrix indicates the local linear subspace tangent to the manifold. The unstable subspace correspond to eigenvalues (Floquet multipliers) greater than one. Stable manifolds may be identified as unstable manifolds of periodic orbits reversed in time. One can compute a numerical approximation to the manifold by numerical integration of points in the local tangent space slightly offset from the periodic orbit.
Some efficiencies in the implementation of our approach depend upon an advantageous choice of parameters to describe each invariant manifold. We generate the manifold from points lying on a linear segment of the (one-dimensional) local tangent space near a given point on the periodic orbit, such that the two ends of the segment are approximately connected by a trajectory near the manifold that winds around the manifold exactly once. This ensures that the propagation of the entire segment constitutes an approximation to the entire manifold, provided the entire segment is sufficiently close to the periodic orbit. When comparing times of flight between two trajectory segments on the same manifold, the difference in offset is accounted for using the linear approximation to the dynamics near the periodic orbit.

The two parameters describing each manifold approximation are thus the offset $\epsilon$ of the starting point along the generating segment, and the integration time $T$ with which the starting point is propagated. Denoting the unstable manifold of the initial orbit by the superscript $^u$ and the stable manifold of the target orbit by the superscript $^s$, the set of combined approximate manifold segments is mapped by the four parameters $\epsilon^u$, $T^u$, $\epsilon^s$, and $T^s$. Variations in $T^u$ and $T^s$ correspond to propagation of manifold segments in time, while variations in $\epsilon^u$ and $\epsilon^s$ correspond to smooth variations of the segments in space.

The existence of a dynamical channel requires that the position components of the corresponding endpoints of the two manifold segments are equal, that is, their difference is zero. Hence the set of such points is the set of global minimizers of this difference in the four-parameter space describing the manifold segments, and the search for dynamical channels can be cast as a multivariate optimization problem in the residual. For smoothness, we actually consider its squared Euclidean norm as the objective in the optimization.

Invariant manifolds in the CR3BP typically exhibit very large variations in curvature with respect to both $\epsilon$ and $T$, which can lead to poor results for many common approaches to global optimization. Moreover, since generic global optimization methods are generally constructed based on isolated independent function evaluations, they fail to allow for the fact that the invariant manifolds are not globally defined, leading to computational inefficiencies. We thus introduce a specialized optimization approach based on adaptive numerical continuation that leverages the special structure of the problem and addresses its inherent computational challenges.

Given a solution to a system of ordinary differential equations with state variables $u$ and scalar parameters $\lambda$, and boundary conditions expressed in terms of these variables, consider the goal of describing the family of solutions that results as one of the parameters $\lambda_i$ is varied. In our approach, the system of differential equations is discretized by adaptive Gaussian orthogonal collocation to form an algebraic system approximating the solution to the boundary value problem. The method of pseudo-arclength continuation$^{42}$ is used to vary the solution according to the parameter $\lambda_i$: variables are incremented to as to approximate a constant step.
Figure 2: Multi-level continuation for manifold intersection detection; (a-e) in each level: continuation paths producing selected example fold points (green), all fold points identified (yellow), overlapping local continuation box boundaries (white); (f) zero-displacement folds representing intersections (yellow)
size in the collocation’s combined extended variable-parameter space. Monitoring the Jacobians of the collocation system allows the solution to detect and continue through folds (extrema) with respect to the parameter \( \lambda_i \).

As the solution is continued in \( \lambda_i \), the collocation mesh is adapted according to the curvature of the local solution. In our application, the pseudo-arclength parameterization adapts manifold segments to changing curvature with respect to \( \epsilon \), and the mesh refinement adapts the segments to changing curvature with respect to \( T_i \), as either parameter is varied. (The latter, combined with sparse linear solvers, may be considered to provide advantages analogous to those of variable step size numerical integration.) This approach thus provides a representation of the segment solution sets for each manifold that is globally adaptive in both time and space, addressing the principal computational difficulties associated with these objects. Since each manifold’s parameters are independent of each other, intelligent caching of intermediate results can allow relatively fast traversals throughout the parameter space.

It is well known that multivariate local extrema can be isolated by the method of successive continuation, here a fold is first located with respect to parameter \( \lambda_1 \); an extended system is formed to continue this fold with respect to the parameter \( \lambda_2 \), simultaneously varying \( \lambda_1 \); and so on, until finding a fold with respect to all the parameters \( \lambda \). However, this method generally fails for global optimization: in particular, while continuation in a single parameter traverses an unbounded sequence of folds, simultaneous continuation of folds in multiple parameters will often traverse a closed loop in parameter space containing only a small number of higher-order folds.

Our approach to global optimization uses continuation of folds to leverage the information about local curvature implicitly provided by the continuation process, thus adapting the search to the complicated geometric structure of the manifolds; however, it applies a multi-layered form of successive continuation in order to ensure a comprehensive exploration of the parameter space. Beginning from a starting point (ideally near the center of the parameter space of interest), the procedure generates candidate multivariate minimizers through a recursive process.

First, each parameter \( \lambda_i \) is assigned an integer layer bound \( b_i \); these are used to limit the recursion depth. For the starting solution, each parameter \( \lambda_i \) is assigned an initial layer count \( c_i = 0 \). For each parameter \( \lambda_i \), the solution is then continued in both directions with respect to \( \lambda_i \) alone, within the extent of the desired parameter space, locating folds in the objective function relative to \( \lambda_i \); the resulting fold points are added to the solutions set with \( c_i = 1 \). For each solution in the solution set, the process is recursively repeated, continuing and locating folds independently in each \( \lambda_i \) (except the immediately preceding one), and incrementing the corresponding layer count \( c_i \). When the layer count \( c_i \) reaches the layer bound \( b_i \), instead of continuing with respect to subsequent parameters independently, the fold located with respect to \( \lambda_i \) is continued in multiple parameters, thus following multivariate folds. In the end, all solutions are multivariate folds in all the \( \lambda_i \), having been effectively locally optimized by successive continuation.

When continuing a solution in the parameter \( \lambda_i \), if the solution has been generated by a sequence of continuations that includes an earlier continuation in \( \lambda_i \), the history of the most recent such continuation is examined to bound the current continuation. Specifically, the current continuation in \( \lambda_i \) is constrained by the values of \( \lambda_i \) at the fold points preceding and succeeding the fold point in the earlier continuation that led to the current solution. We may expect that regions outside this interval will be effectively explored by other candidate solutions generated from other fold points. In this way, the search process is hierarchically localized to successively smaller regions of the parameter space, avoiding duplication of effort across different layers of the recursion.

This layered process adapts to the changing curvature of the solution space by everywhere following the local structure of folds in the objective function, and branching to cover the space before closing in on local extrema. Intuitively, folds in the objective with respect to \( \lambda_i \) at a given point in parameter space separate intervals in which the objective is monotonic in \( \lambda_i \); hence sampling on fold points effectively adapts to the local smoothness of the objective function. Given particular layer bounds \( b_i \), all local extrema are reached by following a sequence of folds from the starting point (including multivariate folds in the final layer) that contain \( b_i \) instances of \( \lambda_i \). The necessary layer bounds depend on the difficulty of the problem and the quality...
of results desired; for simple examples, we have found \( b_i = 2 \) for all parameters to be reasonably effective.

The complexity of the method may be loosely considered multiply exponential in the density of folds with respect to each parameter \( \lambda_i \), with the multipliers \( b_i \); however, since the density of folds varies across the parameter space, this characterization can be misleading. In the end, for given parameters, the method in some sense adapts to the true complexity of the problem, with the bounds \( b_i \) providing a means to avoid unnecessary refinement. In addition, the method is easily parallelized, and the space complexity of the generated solution set can be mitigated by detecting duplicate local extrema as they are located.

To illustrate the method, consider the simplified case of finding dynamical channels between the Earth-Moon \( L_1 \) and \( L_2 \) Lagrange points, which have one-dimensional invariant manifolds (Figure 1). Here the parameters \( \epsilon^u \) and \( \epsilon^s \) are irrelevant, and we need only consider the time parameters \( T^u \) and \( T^s \). Figure 2 illustrates the steps of the global optimization process in these two parameters with bounds \( b_i = 2 \), superimposed on a contour plot of the actual squared distance between the manifolds points determined by each combination of parameter values.

For our purposes, only solutions with zero objective value (position-space intersections of manifold segments) are considered, as they constitute true dynamical channels. Future work will investigate the possibility of constructing candidate transfer solutions from near-intersections using multi-impulse adjoining arcs.

Our implementation makes use of the software package AUTO for continuation and bifurcation analysis of ordinary differential equations. This package automates the adaptive-step pseudo-arclength continuation with adaptive-mesh Gaussian collocation used to overcome the strong nonlinearities in the invariant manifold geometry. To handle two-segment dynamical channels, the equations of motion are effectively duplicated and integrated across the same normalized time interval \([0, 1]\), with the forcing terms scaled by \( T^u \) and \(-T^s\), respectively. Several customizations are used to implement intelligent caching and extraction of intermediate results, so that each branch of the search process may take advantage of portions of the combined manifold space already described.

Improving Solutions by Optimal Control

To locally optimize candidate solutions generated by dynamical channels, we follow the general strategy developed in the work of Lion & Handelsman and Jezewski & Rozendaal. Prior research has used a similar strategy to locally optimize transfer trajectories connecting points on manifolds; in our approach, we apply the optimization to the complete candidate solution generated from each dynamical channel, with endpoints at the corresponding offsets from the initial and target periodic orbits, and an interior impulse at the intersection of the manifold segments.

The primer vector refers to the vector of Lagrange multipliers corresponding to the velocity vector in a standard optimal control formulation. A set of local optimality conditions on the trajectory may be stated in terms of the primer vector:

1. The primer vector and its first derivative are continuous along the entire trajectory;
2. At each propulsive impulse, the primer vector lies in the direction of the impulse with unit magnitude;
3. Everywhere else on the trajectory, the primer vector magnitude does not exceed unity;
4. At each interior impulse, the first derivative of the magnitude of the primer vector is zero.

Given a reference solution, if the primer vector magnitude anywhere exceeds unity, the maximum improvement, to first order, in the trajectory can be achieved by adding an interior impulse at the point at which the primer vector magnitude is maximum, in the direction of the primer vector. The magnitude and timing of the impulse can be subsequently adjusted by iterative root-finding in a small number of parameters to satisfy the optimality conditions. The above procedure may then be iterated, adding an additional impulse at each iteration, until a trajectory is found that satisfies the local optimality conditions.
As in the first stage of our approach, adapting this technique to trajectories constructed from dynamical channels requires careful implementation to avoid numerical instabilities. The algorithm described by Jezewski & Rozendaal\textsuperscript{40} requires computing the state transition matrix associated with the initial (suboptimal) trajectory to solve the boundary-value problem for the primer vector profile along the trajectory. In our applications, this state transition matrix exhibits strongly nonlinear behavior in accordance with that exhibited by the invariant manifolds themselves: for instance, in the example shown below, the maximal Lyapunov exponent of the unstable manifold is on the order of \(~100\) per week of integration time, and thus the matrix norm grows to several orders of magnitude within the examined four-week period.

The state transition matrix is computed by combining the equations of motion with the matrix variational equations of motion, thus computing the trajectory segments and their state transition matrices together in the same integration, and enabling numerical continuation of the entire system. To achieve initial convergence of this extended system, the solution to the variational components is first initialized by computing them, not by numerical integration from their initial conditions (the identity matrix), but using the initial solutions already generated for the trajectory segments. This provides a close enough solution for convergence of the collocation and subsequent continuation, but the speed of convergence can be improved by using the analytic Jacobians of the terms of the state transition matrix, a $36 \times 36$ matrix computed by automatic differentiation.

The trajectory is then modified, first by an initial guess of the optimal magnitude of an additional impulse computed from second-order variations, and subsequently by later estimates generated from a gradient-based optimizer (effectively, numerical root-finding on the objective gradient computed from the profile of the primer vector derivative). However, due to the instability of these trajectories with respect to their boundary conditions, these modifications cannot be accomplished in a single step as they are with Lambert solutions in the two-body case; rather, we again employ successive continuation to successively drive individual terms in the boundary residuals to zero. Again, the time- and space-adaptive nature of the continuation process allows the solution to converge despite the poor numerical behavior of the invariant manifolds. Results from this second stage of our approach will be described in a follow-on paper.

RESULTS

As an illustrative example, we consider a planar transfer between two Lyapunov libration orbits about the $L_1$ and $L_2$ Lagrange points (respectively) of the Earth-Moon system. These Lyapunov orbits are representative of orbits considered for libration point mission applications, and offer convenient transfers to orbits of interest in the Earth and Moon neighborhoods. The initial orbit is an $L_1$ Lyapunov orbit with Jacobi constant $C = 3.196$, while the target orbit is an $L_2$ Lyapunov orbit with Jacobi constant $C = 3.178$. As these two constants are unequal, no nonpropulsive transfer exists between these two orbits.

Figure 3 shows portions of the unstable manifold of the $L_1$ orbit and stable manifold of the $L_2$ orbit in the region of interest, each propagated out to roughly 4 weeks in duration. Beyond this time interval, the manifolds depart close proximity for a much longer time, hence this is a reasonable interval to examine for this problem. The depiction of the manifold segments in Figure 1 is generated by the same adaptive numerical continuation used in our global optimization procedure for locating dynamical channels: it thus gives a visual indication of how the sampling globally adapts to changes in local curvature everywhere on the manifold.

Figure 4 shows the most efficient solutions generated from dynamical channels in the first stage of our method. It is clear that the global optimization procedure explores several distinct families of solutions, some with sufficiently dense sampling that a local Pareto front of single-impulse trajectories is visible. In this example, the efficient trajectories among these candidate solutions with the lowest range of $\Delta V$ (for instance, Figure 5(a) and (b)) all belong to a single family. A few isolated intersections allow transfers with substantially lower TOF at a cost of greater $\Delta V$ (Figure 5(c) and (d)); these have a similar geometry, but transfer earlier to the faster-moving stable manifold of the target orbit. The sampling here is more sparse, since the individual manifolds behave in a relatively smooth fashion: future work will explore the possibility of adapting the continuation to the relative local geometry of the manifolds in their combined parameter space, rather than their individual absolute geometry. However, we may expect that, since this entire region of solutions employs a similar geometry, subsequent multiobjective optimization can successfully fill out the
Figure 3: (a) Unstable and (b) stable manifolds of initial and target Lyapunov orbits sampled by pseudo-arclength continuation in $\epsilon^u$ and $\epsilon^s$, respectively; (c) overlay of (a-b): intersections form dynamical channels
Figure 4: Portion of locally optimal transfer solutions generated

Figure 5: (a-d) Sample efficient trajectories from the Pareto front with increasing $\Delta V$ and decreasing TOF
Figure 6: An example trajectory (not Pareto-optimal) generated from different branch of dynamical channels.

Other families of dynamical channels produce transfers with substantially different geometry: for instance, Figure 6, generated from the thin family in the right half of Figure 4. While the candidate solutions generated from the dynamical channels in this family are not Pareto-optimal, there is certainly the potential for efficient solutions to be generated through local optimization. A follow-on paper will explore more complex problems (such as three-dimensional transfer problems), that may exhibit contributions to the Pareto front from many such families of dynamical channels: in a situation like this, a global optimization approach like the one described here can be essential to developing a comprehensive set of options for mission design.

CONCLUSION

We have described a two-stage approach to constructing comprehensive multiobjective-optimal solution sets for low-energy transfer problems between arbitrary unstable periodic orbits, an important capability to support trajectory design for missions leveraging multi-body dynamics to reduce fuel requirements. A specialized adaptive approach to global optimization is used to identify position-space intersections of invariant manifolds forming dynamical channels, from which candidate solutions may be locally improved using optimal control to generate an approximation to the Pareto front over parameter ranges of interest. In this paper, the first stage of the method is demonstrated to produce a range of potentially useful candidate solutions with distinct geometries in an example libration point orbit transfer application. A follow-on paper will demonstrate results of the second stage of the approach involving local optimization of these candidate solutions, and explore examples of more complex transfer problems pertinent to present applications.

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