The Arbitrated Network Control Systems Approach to CPS

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Outline

• Networked Control Systems (NCS)
  – Definition
  – Challenges
  – Current Solutions

• Arbitrated NCS (ANCS)
  – Definition
  – Need for an ANCS
  – Implementation in ANCS
  – Hierarchical ANCS
  – Multi-modal ANCS
NETWORKED CONTROL SYSTEMS
Traditional Control System

- Centralized System
- Analog signal transfer
- Point-to-point communication
- One wire per signal
- Ideal signal transfer assumed
A Networked Control System is a spatially distributed control system where information is exchanged over a (digital) network.
Features of NCS

- Sensors and controllers can be added or removed without wiring efforts
- Increased re-configurability
- Simplification of diagnosis procedures and maintenance
- Hence, reduction of cost
- Efficient sharing of data via network
Advantages of NCS

- Reduced complexity, wiring, and cost of system
- Easy maintenance, diagnosis, and reconfiguration
- Increased flexibility and autonomy
Applications of NCS

- Automobile industry in 1970's
  - Driven by reduced cost for cabling, modularization of systems, and flexibility in car manufacturing
- A wide range of applications at present

- Engineering Networks
  - Manufacturing automation
  - Automotive Systems
  - Aircraft
  - Teleoperation & Remote Surgery
  - Building automation
  - Automated highway systems
  - Environmental monitoring and control

- Physical/biological/ecological networks
  - Synchronization networks
  - Flock of birds/school of fish
  - Gene/cell networks
  - Food webs
  - Social networks
Example 1: Automotive Systems

- ~100 control units
  - Engine Control
  - Idle speed control
  - Drive by wire
  - Lights
  - Diagnosis
  - Cruise Control
Example 2: Manufacturing

- Example of complex process control

Highly interconnected control systems in a manufacturing process

An industrial bus protocol: Profibus
Example 3: Power Networks

- Heterogeneous power generation networks
  - Many small power plants connected:
    - Solar
    - Wind mill
    - Nuclear power plants
    - Gas turbines
    - ....
Example 4: Traffic Management

- Motivation:
  - Increase traffic throughput
  - Avoid congestion
- Install wireless traffic sensors
- Model traffic as partial differential equations
Highlights of NCS Solutions

- Analysis and synthesis of controllers that are robust to
  (1) delays,
  (2) varying delays,
  (3) packet dropouts

Some examples of solutions are discussed here.
Control designs that are robust to constant delay

\[ \dot{x} = Ax + B\hat{y}, \quad y = Cx \]

\[ \hat{y}_k = y_k \quad \forall k \in \mathbb{N} \]

\[ \hat{y}(t) = \begin{cases} \hat{y}_{k-1}, & t \in [t_k, t_k + \tau_k) \\ \hat{y}_k, & t \in [t_k + \tau_k, t_{k+1}) \end{cases} \]

**Theorem:**
Assuming that there exist constants \( h > \tau \geq 0 \) such that

\[ t_{k+1} - t_k = h, \quad \tau_k = \tau, \quad \forall k \in \mathbb{N} \]

the NCS (1)-(3) in the Figure above is exponentially stable if and only if \( \Phi(h, \tau) \) is Schur.

Imposes limits on the delay for satisfactory behavior

Control designs that are robust to variable delay\(^{(2)}\)

\[\xi_{k+1} = \bar{A}(\tau_k)\xi_k, \quad \tau_k \in [0, \tau_{\text{max}}]\]

with

\[\bar{A}(\tau_k) = \begin{bmatrix} e^{Ah} - \Gamma_0(\tau_k)BK & -\Gamma_1BK \\ I & 0 \end{bmatrix}\]

and \[\xi_k = (x_k^T \quad x_{k-1}^T)^T\]

**Theorem:**

Given the system above with the delay-dependent matrix \[\bar{A}(\tau_k)\]. If there exists a solution to the discrete-time Lyapunov matrix inequalities

\[P = P^T > 0\]

\[\bar{A}^T P \bar{A} - P < 0, \quad \forall \bar{A} \in \mathcal{A}\]

for a suitable chosen finite set of matrices \[\mathcal{A}\], then the system is robustly globally asymptotically stable for any sequence of delays \[\tau_k \in [0, \tau_{\text{max}}]\]
Control designs that are robust to packet dropouts

$$z_{k+1} = \Phi_\theta z_k$$

where

$$\Phi_\theta = \begin{bmatrix} e^{Ah} + \theta \Gamma(h - \tau)BC & e^{A(h-\tau)}\Gamma(\tau)B + (1 - \theta)\Gamma(h - \tau)B \\ \theta C & (1 - \theta)I \end{bmatrix}$$

for $\theta \in 0, 1$

Theorem:

The NCS given above with dropout probability $p$ (Bernoulli) is exponentially mean-square stable if there exists a symmetric matrix $Z > 0$ such that

$$\begin{bmatrix} Z & \sqrt{p}(\Phi_0 Z)' & \sqrt{1 - p}(\Phi_1 Z)' \\ * & Z & 0 \\ * & * & Z \end{bmatrix} > 0$$

ARBITRATED NETWORKED CONTROL SYSTEMS (ANCS)
Structure of communication networks

- Two groups of communication networks:
  - Data networks
    - Example: Internet
    - Large data packets and high throughput
    - Delays and packet dropouts non-deterministic
  - Control networks
    - Example: Brake control in an automobile
    - Small data packets at a high frequency to meet real-time requirements
    - Transmission delays are deterministic or at least bounded

- Distinction between data and control networks:
  - Network protocol capable of supporting real-time or time-critical applications
Structure of communication networks (2)

- Two groups of transmission technology
  - Wired
    - Large bandwidth, high reliability, good security
    - Limited flexibility and mobility
  - Wireless
    - Outstanding flexibility and mobility
    - Limited bandwidth, less reliable transmission
Examples of Protocols in Wired and Wireless Networks

- **Wired networks:**
  - Token-passing bus (Media Access Control Sub Layer)
  - Controller Area Network (CAN)
  - Ethernet (Hub-based or Switch-based)
  - Internet Protocol (IPv4, IPv6)
  - Transmission control protocol (TCP)
  - User Datagram Protocol (UDP)

- **Wireless networks**
  - Wireless Ethernet

Certain choices exist for the network protocol, with different properties
Our focus: Arbitrated NCS (ANCS) (1)

• What is ANCS?
  – Subset of NCS
  – Wired
  – Spatially localized (ex. an automobile, a building, a factory)
  – Dedicated network
  – Shared network
  – Fault-tolerant
  – Makes use of the protocol choices

• Why “Arbitrated”?
  – The network has to service several applications
  – Limited resources
  – Messages have to be scheduled judiciously, i.e. _arbitrated_
The Arbitrated NCS (ANCS) (2)

• Synthesis of ANCS
  – Utilize the structure of the communication network
  – Choose the protocol in the different OSI layers
  – Optimize the protocol parameters
  – Optimize the network architecture
  – Optimize the controller design
  – Co-design the architecture together with the controller
Goal of an ANCS

- Optimize the network architecture based on constraints coming from the control applications
- Optimize the control design based on constraints coming from the network structure (protocol, topology, etc.)
- Co-design both control and the network
A Computer-Controlled System

\[ x[k + 1] = Ax[k] + Bu[k] \]
\[ \dot{x}(t) = Ax(t) + Bu(t) \]

Control Implementation in A DES

DES: Distributed Embedded System

\[ u = f(x) \]

Actuator

Plant

Sensor

Processing Unit

Actuator Task

Controller Task

Sensor Task

Control Implementation in A DES (contd.)

\[ u = f(x) \]

Actuator

Scheduler

\( T_a \)

PU1

\( T_c \)

Sensor

Scheduler

\( T_s \)

PU2

\( x \)

Complex structure of the CPS necessitates arbitration
Two main features of ANCS

- Hierarchy
- Multi-modality
HIERARCHICAL ANCS
Problem Statement:

Plant: \[ \dot{x}(t) = Ax(t) + Bu(t) \]

Sampling Period: \( h \)

Sampled Plant-model:
\[ x(kh + h) = \Phi x(kh) + \Gamma_1 u(kh - h) + \Gamma_0 u(kh) \]
Hierarchical ANCS

- Structure of the Delay in an ANCS
  - Parts of the delay are known!
- $\tau_{\text{known}}$ can be used in the Control Design
- $\tau_{\text{unknown}}$ needs to be adapted to
- Sampled Plant-model:

$$x(k+1) = e^{A_l s} x(k) + \int_{\sigma_u + k l s + \tau_{ca}}^{\sigma_x + (k+1) l s} e^{A(\sigma_x + (k+1) l s - \xi)} d\xi B u(\sigma_u + (k - 1) l s)$$

$$+ \int_{\sigma_u + k l s}^{\sigma_x + (k+1) l s} e^{A(\sigma_x + (k+1) l s - \xi)} d\xi B u(\sigma_u + k l s)$$
Hierarchical ANCS

The Adaptive Controller

Plant-model: \( x(k + 1) = \Phi x(k) + \Gamma_0 u(k - 1) + \Gamma_1 u(k) \)
\[ = p^T w(k) + \Gamma_1 u(k) \]

If \( \tau_{\text{unknown}} \) is known, choose controller
\[ u(k) = -p_1^T w(k) + p_2^* r(k) \]
with \( p_1^* = \Gamma_1^{-1} p^T \) and \( p_2^* = \Gamma_1^{-1} \)

\( \tau_{\text{unknown}} \) is unknown, choose Adaptive Controller:
\[ u(k) = -\hat{p}_1^T(k) w(k) + \hat{p}_2(k) r(k) \]
\[ \hat{p}_1(k + 1) = \hat{p}_1(k) - \frac{\gamma_1 \text{sgn}(\Gamma_1) e(k + 1) w(k)}{1 + w^T(k) w(k) + r^2(k)} \]
\[ \hat{p}_2(k + 1) = \hat{p}_2(k) - \frac{\gamma_2 \text{sgn}(\Gamma_1) e(k + 1) r(k)}{1 + w^T(k) w(k) + r^2(k)} \]
Proof of Stability

Consider the positive function

\[ V(k) = |\Gamma_1| \gamma_1^{-1} \tilde{p}_1^T(k) \tilde{p}_1(k) + |\Gamma_1| \gamma_2^{-1} \tilde{p}_2^2. \]

Then \( \Delta V(k) = V(k + 1) - V(k) \)

\[ = \frac{e^2(k + 1)}{m^2(k)} \cdot \left( \frac{|\Gamma_1| \gamma_1 w^T(k) w(k) + |\Gamma_1| \gamma_2 r^2(k)}{m^2(k)} - 2 \right) \leq 0 \]

The Key-technical Lemma 6.2.1 in (Goodwin and Sin, 1984) can be used to show the boundedness of all signals in the system.
Consider the scalar system:  \( \dot{x}(t) = 2x(t) + u(t) \)
Sampling rate changes from 1ms to 10ms

Unable to track reference after change in sampling time
MULTI-MODAL ANCS
Modes in a Control System & a DES

- **Control Systems**: An Example: Transient & Steady-state modes
- **DES**: Static (Time-triggered) and Dynamic (Event-triggered) Segments

![Diagram showing communication cycles and static/dynamic segments](image-url)
Multi-modal Co-design: An Example

Co-design Strategy:

- **TT communication** is suitable for **transient phase**.
- **ET communication** for the control signals is sufficient for the **steady-state phase**.

*D. Goswami, R. Schneider and S. Chakraborty, "Re-engineering Cyber-Physical Control Applications for Hybrid Communication Protocols", DATE, 2011.*
Problem

- Control of multiple applications using a DES

\[ C_i : x[k + 1] = A_1 x[k] + B_1 u[k] \]
\[ C_j : x[k + 1] = A_2 x[k] + B_2 u[k] \]

\[ A_i = \begin{bmatrix} 0.4 & 1.0 \\ -1.56 & -0.9 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, \]
\[ A_j = \begin{bmatrix} 1.2 & 0.2 \\ -1.8 & -2.1 \end{bmatrix}, \quad B_j = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix} \]

- Suppose an Optimal Controller is designed:
  \[ u[k] = K x[k]; \quad K \text{ determined using an LQG-LTR method} \]
Overall Goal

**Hybrid Communication Protocol**
(1) Time-triggered Segment: Predictable and expensive
(2) Event-triggered Segment: Unpredictable and less expensive

**Performance of Distributed Control Applications**
(1) Very good with time-triggered communication
(2) Unacceptable with event-triggered communication

Switching scheme between control strategy and communication segment (time- and event-triggered)

Trade-off between control performance and utilization of the time-triggered segment (cost of communication)
Multi-modal Co-Design

**Mode Switching**

**TT:**
- Schedule:
- Controller:
- Plant in Steady-State

**ET:**
- Schedule:
- Controller:
- Plant in Steady-State

**Mode Change Protocol**

- Plant in Transient

**After dwell time**

- After dwell time

- Disturbance

\[ \xi_i \]
Simulation Results

- **Pure Time-Triggered**
  - Fast response
  - with a more expensive Communication protocol

- **Pure Event-Triggered**
  - Slow response with a cheaper protocol

- **Switching**
  - A trade-off between control performance and cost of communication

Leads to a switching control strategy – needs to be shown to be stable!
Problem in the Presence of Uncertainties

- **Plant Dynamics:**
\[
\begin{align*}
    x_i(k+1) &= A_i x(k) + \lambda_i b_i u_i(k) \\
    y_i(k) &= c_i^T x_i(k) \\
\end{align*}
\]

- **Rewritten as**
\[
y(k) = \theta^*^T \Phi(k-1) + \lambda b_1 u(k-1)
\]

- **Baseline Control:**
\[
u(k-1) = (\lambda b_1)^{-1} \left( y_d(k) - \theta^*^T \Phi(k-1) \right)
\]

If $\theta^*$ is unknown, an adaptive solution is needed.
Switching Adaptive Controller

• Time-triggered (static): If a slot is available, delay = 0
  \[ y(k) = \theta^T \Phi(k - 1) + \lambda b_1 u(k - 1) \]

• Event-triggered: Delay is present. Let delay = 1

The switching adaptive controller has to be designed to be stable!
Switching Adaptive Controller

\[
\begin{align*}
    u(k) &= \frac{1}{\hat{\beta}_0(k)} \left( y_{\text{ref}}(k + 1) - \bar{\theta}_1(k)^T \Phi_1(k) \right) \\
    \varepsilon_1(k) &= y(k) - \hat{\theta}_1(k - 1)^T \Psi_1(k - 1) \\
    \hat{\theta}_1(k) &= [\hat{\theta}_1(k)^T \hat{\beta}_0(k)]^T \\
    \Phi_1(k) &= \begin{bmatrix} y(k) & \ldots & y(k - m_1 + 1) & u(k - 1) & \ldots & u(k - m_2) \end{bmatrix}^T \\
    \Psi_1(k - 1) &= [\Phi_1(k - 1)^T u(k - 1)]^T \\
    \hat{\theta}_1(k) &= \hat{\theta}_1(k - 1) + \frac{\gamma_1 \Psi_1(k - 1) \varepsilon_1(k)}{1 + \Psi_1(k - 1)^T \Psi_1(k - 1)} \\
    \end{align*}
\]

\[
\begin{align*}
    u(k) &= \frac{1}{\hat{\beta}_0(k)} \left( y_{\text{ref}}(k + 2) - \bar{\theta}_2(k)^T \Phi_2(k) \right) \\
    \varepsilon_2(k) &= y(k) - \hat{\theta}_2(k - 1)^T \Psi_2(k - 2) \\
    \hat{\theta}_2(k) &= [\hat{\theta}_2(k)^T \hat{\beta}_0(k)]^T \\
    \Phi_2(k) &= \begin{bmatrix} y(k) & \ldots & y(k - m_1 + 1) & u(k - 1) & \ldots & u(k - m_2 - 1) \end{bmatrix}^T \\
    \Psi_2(k - 2) &= [\Phi_2(k - 2)^T u(k - 2)]^T \\
    \hat{\theta}_2(k) &= \hat{\theta}_2(k - 1) + \frac{\gamma_2 \Psi_2(k - 2) \varepsilon_2(k)}{1 + \Psi_2(k - 2)^T \Psi_2(k - 2)} \\
\end{align*}
\]

if \( \mathcal{M}_{\text{Bus}}(k) = M_{TT} \)  

if \( \mathcal{M}_{\text{Bus}}(k) = M_{ET} \)
Main Theorem

**Theorem:**
Under the switching algorithm in (1) and (2), there exists a \( T_{dw, \text{min}} \) such that for all dwell times \( T_{dw} \geq T_{dw, \text{min}} \) the overall adaptive system is globally asymptotically stable and the tracking error \( e(k) = y(k) - y_{\text{ref}}(k) \) and the parameter error \( \tilde{\theta} \) are bounded.
Outline of Proof

Let there exist a sequence of finite switching times \( \{k_l\}_{l \in \mathbb{N}} \)

For \( k \in [k_{2p} + 1; k_{2p+1}], p \in \mathbb{N}_0: \mathcal{M}_{\text{Bus}}(k) = M_{\text{TT}} \)

For \( k \in [k_{2p+1} + 1; k_{2p}], p \in \mathbb{N}_0: \mathcal{M}_{\text{Bus}}(k) = M_{\text{ET}}. \) Let \( M_{\text{Bus}}(k) \in TT \) for \( k \in [k_0 + 1, k_1] \)

**Step 1:** There exists a \( T_{dw} \) such that \( |e_1(k_1)| < \varepsilon \leq e_{\text{th}} \) where \( k_1 = k_0 + T_{dw} \)

At \( k_1' := k_1 + 1 \), Protocol switches to ET; Delay increases from 0 to 1; There is a jump in the output error;

**Step 2:** \( |e(k_1')| \leq |e(k_1)| + M \) \( M \) is finite

For \( \varepsilon = e_{\text{th}} - M \), it follows that \( \mathcal{M}_{\text{Bus}}(k_1') = M_{\text{ET}}. \) For \( [k_1', k_2], M_{\text{Bus}}(k) \in M_{\text{ET}} \)

At \( k_2' := k_2 + 1 \), Protocol switches to TT; Delay decreases from 1 to 0; There is a jump in the output error;

**Step 3:** \( |e(k_2')| \leq |e(k_2)| + M \); Since \( M_{\text{Bus}}(k_2) = M_{\text{ET}}, \)

\( |e(k)| \leq e_{\text{th}} + M \) for all \( k \in [k_2'; k_3], \) i.e., as long as the system is in \( M_{\text{TT}} \)

\( k_0, k_1, k_2 \) are arbitrary; This proves that \( e(k) \) is bounded.

• In each of the three steps above, \( V(k) = \tilde{\theta}_a(k)^T \tilde{\theta}_a(k) \) can be shown to be bounded as well using similar arguments (i.e. bounded in TT and ET, with a finite jump at switches)

• Key Technical Lemma in Goodwin & Sin ensures boundedness of all other signals
Publications


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- H. Voit, A. Annaswamy, Adaptive Control of a Networked Control System with Hierarchical Scheduling, American Control Conference (ACC), San Francisco, California, USA, 2011


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