Recitation 10

We covered Lagrange Multipliers and some two, nonindep. vars

I did not get to this.

Idea: want to solve problems like

"Maximize \( z = f(x,y) \) subject to constraint \( g(x,y) = 0 \)"

(2D version)

Have two choices
- substitute \( g \) into \( h \) or something like this
- use a clever trick:

Note that because of continuity, the max \( z = f(x^*,y^*) \) will

occur at \( p = (x^*,y^*) \) where the level curve \( f(x,y) = z^* \)

is parallel to the constraint curve \( C : g(x,y) = 0 \)

(tangent to)

But at point where \( g = 0 \), \( z^* = f(x^*,y^*) \) are tangent, the

gradients of \( g(x,y) \) and \( f(x,y) \) must be collinear - i.e., must be

multiples of each other. So find the pt by solving

\[
\nabla f = \lambda \nabla g
\]

Equivalently, write

\[
L = f(x,y) - \lambda g(x,y)
\]

and take partials, set all equal to \( 0 \):

\[
\begin{align*}
\frac{\partial L}{\partial x} &= 0 \\
\frac{\partial L}{\partial y} &= 0 \\
\frac{\partial L}{\partial \lambda} &= 0
\end{align*}
\]

Examples:

Talked about the examples in the book, chap. 19:

What is the point on the plane \( P \)

\( x + 2y + 3z = 0 \)

closest to the origin.

(Mint: minimize distance squared)