Review double integrals, some problems.

Integration in polar coordinates: Old variables x, y, New variables r, \( \theta \).

2. Change area unit: \( dA_{\text{Polar}} = r\,dr\,d\theta \).
3. Change integrated function.

Useful conversion formulas:

- \( x = r \cos \theta \), \( y = r \sin \theta \)
- \( r^2 = x^2 + y^2 \)
- \( \cos \theta = x/r \)
- \( \sin \theta = y/r \)
- Pythagorean Thm: \( r^2 = x^2 + y^2 \)

Evaluate:

(a) \( \iint_{D} 2xy \, dA \) D - portion between \( A, A', B_2, B_5 \), first quadrant.

We should get 609/4.

(b) Determine area inside \( r = 3 + 2 \sin \theta \), outside \( r = 2 \).

\( 3 + 2 \sin \theta = 2 \quad \Rightarrow \quad \theta = \frac{3\pi}{6}, \frac{11\pi}{6}, \left(-\frac{\pi}{6}\right) \)

\[ A = \iint_{D} dA = \text{---} \]

\[ = \int_{-\pi/6}^{7\pi/6} \left(\frac{7}{2} + 6 \sin \theta - \cos 2\theta\right) d\theta = \frac{11\sqrt{3}}{2} + \frac{14\pi}{3} \]
Determine volume under \( x^2 + y^2 + z^2 = 9 \) above \( z = 0 \) inside \( x^2 + y^2 = 5 \)

\[
\int_0^{\pi/2} \int_0^5 r^2 \, dr \, d\theta = \int_0^{\pi/2} \frac{9\pi}{3} \, d\theta = \frac{38\pi}{3}
\]

The above three problems all come from one tutorial.

The one we solved in detail in class was the first one.

We did two things

1) Basic evaluation of double integrals
   - Do inside first, treating other (outside) variable as constant
   - Then evaluate outside integral as a simple one-dim (one var.) integral

\[
\int_0^3 \int_0^{3x^2} xy \, dy \, dx
\]

\[
\int_0^3 \int_0^{3x^2} xy \, dy \, dx = \int_0^3 \left[ \frac{x^2 y^2}{2} \right]_0^{3x^2} \, dx = \int_0^3 \left( \frac{9x^6}{2} - \frac{x^4}{2} \right) \, dx
\]

\[
= \int_0^3 \frac{x^6}{2} - \frac{x^4}{2} \, dx = \frac{x^7}{12} - \frac{x^5}{8} \bigg|_0^3 = \frac{3^7}{12} - \frac{3^5}{8} = 81 - \frac{243}{8} = \frac{81 \cdot 8}{8} - \frac{243}{8} = \frac{648 - 243}{8} = \frac{405}{8}
\]

(2) Evaluated #1 from above

D: \[ 2 \leq r \leq 5 \quad 0 \leq \theta \leq \frac{\pi}{2} \]

\[
\iint_D 2xy \, dA = \int_0^{\pi/2} \int_0^5 2rcos\theta \sin^3 \theta \, d\theta \, dr = \int_0^{\pi/2} \int_0^5 2r^3 \cos \theta \sin^3 \theta \, d\theta \, dr
\]

(see Paul's notes for details) = evaluate integral here = \( \frac{609}{4} \)