Summary of Recalculation 9

Question: What is the rate of change of function in this specific direction?

Answer: The derivative \( f_u = \nabla f \cdot \vec{u} = f_x u_x + f_y u_y \).

Some problems we did:

\[ f(x, y) = x^2 y^3 \quad P = (-1, 2) \]

Find vector in direction of:
- Max rate of change (a)
- Min rate of change (b)
- Zero rate of change (c)

\[ \nabla f = (2xy^3, 3x^2y^2) \]

\[ \nabla f(-1, 2) = (-16, 12) \]

Let \( \nabla f = (u, v) \)

Then \( u_1 = (5, 5) - (1, 1) = (3, 4) \)

\[ u_2 = (1, 3) - (0, 1) = (1, 2) \]

\[ f_{u_1} = 1 \quad \Rightarrow \quad \frac{3a + \frac{5}{2}b}{\sqrt{5}} = 1 \]

\[ f_{u_2} = -\frac{2}{\sqrt{5}} \quad \Rightarrow \quad \frac{\frac{1}{\sqrt{5}} - 1}{1} a + \frac{1}{\sqrt{5}} (2) b = -\frac{2}{\sqrt{5}} \]

Solve:

\[ \begin{cases} 3a + 4b = 5 \\ -a + 2b = -2 \end{cases} \]

\[ a = \frac{18}{10}, \quad b = -\frac{1}{10} \]

\[ \frac{\partial z}{\partial x} = \frac{18}{10}, \quad \frac{\partial z}{\partial y} = -\frac{1}{10} \]

Notes on Chain Rule:

Matrix notation:

\[ z = f(x, y) \]

Then we can write ordinary partials in matrix format:

\[ \begin{bmatrix} \frac{\partial z}{\partial u} \\ \frac{\partial z}{\partial v} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{bmatrix} \begin{bmatrix} \frac{\partial z}{\partial x} \\ \frac{\partial z}{\partial y} \end{bmatrix} \]

Do you see a pattern here? How are these matrices constructed?