PROBLEM 13.29

**KNOWN:** Two parallel plates approximating blackbodies located in surroundings at a prescribed temperature.

**FIND:** Net heat transfer by radiation *from* each plate and *to* the surroundings.

**SCHEMATIC:**

![Schematic Diagram](image_url)

- $A_1, T_1 = 750K$
- $T_3 = 300K$
- $A_2, T_2 = 500K$

**ASSUMPTIONS:** (1) Surfaces behave as blackbodies, (2) Surroundings are large compared to plates, (3) Backsides of plates are insulated.

**ANALYSIS:** The general expression for net transfer of radiation *from* a black surface due to exchange with other black surfaces is given by Eq. 13.14. For three surfaces,

$$ q_i = A_i \sum_{j,k} F_{ij} \sigma (T_j^4 - T_i^4) + A_i \sum_{j,k} \sigma (T_k^4 - T_i^4) $$

where $i, j, k$ represent surfaces 1,2,3. From Fig. 13.4, find that $F_{12} = 0.2$ for $X/L = 1$ and $Y/L = 1$. Note also $F_{21} = F_{12}$. From the summation rule for surface $A_1$, $F_{13} = 1 - F_{12} = 1 - 0.2 = 0.8$. Note also that $F_{13} = F_{23}$. The net radiation heat transfer *from* surface $A_1$ is then

$$ q_1 = A_1 F_{12} \sigma (T_2^4 - T_1^4) + A_1 F_{13} \sigma (T_3^4 - T_1^4) $$

$$ q_1 = (1 \times 1) m^2 \times 0.2 \times 5.67 \times 10^{-8} W/m^2 K^4 (750^4 - 500^4) K^4 + $$

$$ (1 \times 1) m^2 \times 0.8 \times 5.67 \times 10^{-8} W/m^2 K^4 (750^4 - 300^4) K^4 $$

$$ q_1 = 2879 W + 13,985 W = 16,864 W. $$

The net radiation heat transfer *from* surface $A_2$ is then

$$ q_2 = A_2 F_{21} \sigma (T_2^4 - T_1^4) + A_2 F_{23} \sigma (T_3^4 - T_2^4) $$

$$ q_2 = (1 \times 1) m^2 \times 0.2 \times 5.67 \times 10^{-8} W/m^2 K^4 (500^4 - 750^4) K^4 + $$

$$ (1 \times 1) m^2 \times 0.8 \times 5.67 \times 10^{-8} W/m^2 K^4 (500^4 - 300^4) K^4 $$

$$ q_2 = -2879 + 2,468 = -411 W. $$

The net radiation heat transfer *from* the surroundings, surface $A_3$ is written as

$$ q_3 = A_3 F_{31} \sigma (T_3^4 - T_1^4) + A_3 F_{32} \sigma (T_2^4 - T_3^4) = A_1 F_{13} \sigma (T_3^4 - T_1^4) + A_2 F_{23} \sigma (T_3^4 - T_2^4) $$

$$ q_3 = (1 \times 1) m^2 \times 0.8 \times \sigma (300^4 - 750^4) W/m^2 + (1 \times 1) m^2 \times 0.8 \times \sigma (300^4 - 500^4) W/m^2 = -16,452 W. $$

Note the use of the reciprocity rule. It follows the net heat transfer by radiation to the walls or surroundings is

$$ q_{net, sur} = +16,452 W. $$

Note that $q_1 + q_2 + q_3 = 0$. 

△
PROBLEM 13.46

KNOWN: Long, thin-walled horizontal tube with radiation shield having an air gap of 10 mm. Emissivities and temperatures of surfaces are prescribed.

FIND: Radiant heat transfer from the tube per unit length.

SCHEMATIC:

\[ T_1 = 120^\circ C \quad T_2 = 35^\circ C \quad \text{Air gap} \]

\[ \text{Shield, } D_2 = 120 \text{ mm, } \epsilon_2 = 0.1 \]

\[ \text{Tube, } D_1 = 100 \text{ mm, } \epsilon_1 = 0.8 \]

ASSUMPTIONS: (1) Tube and shield are very long, (2) Surfaces at uniform temperatures, (3) Surfaces are diffuse-gray.

ANALYSIS: The long tube and shield form a two surface enclosure, and since the surfaces are diffuse-gray, the radiant heat transfer from the tube, according to Eq. 13-23, is

\[ q_{12} = \frac{\sigma (T_1^4 - T_2^4)}{1 - \epsilon_1} \frac{1}{A_1} + \frac{1 - \epsilon_2}{\epsilon_2 A_2} \]

(1)

By inspection, \( F_{12} = 1 \). Note that

\[ A_1 = \pi D_1 \ell \quad \text{and} \quad A_2 = \pi D_2 \ell \]

where \( \ell \) is the length of the tube and shield. Dividing Eq. (1) by \( \ell \), find the heat rate per unit length,

\[ q'_{12} = \frac{q_{12}}{\ell} = \frac{5.67 \times 10^{-8} \text{ W/m}^2\text{K} \left[ (273 + 120)^4 - (273 + 35)^4 \right]}{0.8 \pi (100 \times 10^{-3} \text{ m})} + \frac{1}{\pi (100 \times 10^{-3} \text{ m}) \times 1} + \frac{1 - 0.1}{0.1 \pi (120 \times 10^{-3} \text{ m})} \]

\[ q'_{12} = \frac{842.3 \text{ W/m}^2}{(0.7958 + 3.183 + 23.87) \text{m}^{-1}} = 30.2 \text{ W/m}. \]

COMMENT: Recognize that convective heat transfer would be important in this annular air gap. Suitable correlations to estimate the heat transfer coefficient are given in Chapter 9.