Blackbody (Ideal surface)

1. Absorbs all incident radiation
   \[ \alpha = 1 \]

2. Is an ideal emitter
   \[ \varepsilon = 1 \]

3. Emitted radiation is diffuse (independent of direction)

Gray surface

1. \( \alpha \) and \( \varepsilon \) are independent of wave length

2. \( \alpha = \varepsilon \)
Radiation Exchange

\[ J \]

\[ \alpha G \]

\[ 2G \]

\[ p + \alpha + 2 = 1 \quad \text{for opaque material} \quad 2 = 0 \]

Net radiation exchange at one surface

\[ q_i = A_i (J_i - G_i) \]

\[ = A_i (E_i - \alpha G_i) = \frac{E_i - J_i}{(1 - \alpha) / \alpha A_i} \leftarrow \text{Resistance} \]

\[ G = E_{\text{sur}} = \nabla T_{\text{sur}}^4 \]

\[ \alpha = \varepsilon \quad (\text{Gray surface}) \]

\[ \delta_{\text{rad}} = A_i (\varepsilon E_{\text{sur}} - \varepsilon E_{\text{sur}}) \]

\[ = A \cdot \varepsilon \nabla T_s \quad (T_s - T_{\text{sur}}) \Rightarrow h_r \]

Radiation exchange between surfaces

* View Factors:
  1. Reciprocity relation: \( A_i F_{ij} = A_j F_{ji} \)
  2. Summation rule: \( \sum_{j=1}^{n} F_{ij} = 1 \)

Table 13.1: View factor for 2-D geometries
Table 13.2: 3-D geometries

Fig 13.4-13.6: VF for common geometries
Two-Surface Endorses

General analysis:

\[ q_1 - \frac{E_{b1} - J_1}{1 - \varepsilon_1 z_A} \rightarrow q_{12} \rightarrow J_2 - \frac{E_{b2}}{1 - \varepsilon_2 z_A} \rightarrow -q_2 \]

\[ \varepsilon_1 = \frac{q_{12}}{q_1} = -q_2 = \frac{T (T_2 - T_1^4)}{1 - \varepsilon_1} + \frac{1}{\varepsilon_1 A_1} + \frac{1 - \varepsilon_1}{\varepsilon_2 A_2} \]

Specifically, small convex object in a large cavity

\[ \frac{A_1}{A} \alpha = 1, \quad T_{12} = 1 \]

\[ q_{12} = \tau A_{12} (T_2 - T_1^4) \]

Radiation Shield (13.3.4)

A highly radiative, low \( \varepsilon \) surface between two other surfaces

Analysis doesn't change!

\[ \begin{align*}
q_1 & \rightarrow q_{12} & \rightarrow q_2 \\
& 1 & 3 & 2
\end{align*} \]
chap 6 → fundamentals/physical mechanisms
(Use the example of external flow)
chap 7 → External flow
chap 8 → Internal flow
  How to quantify convective heat flow
  
  \[ \text{determine } h \text{ and } \bar{h} \]
  \[ \text{determine } N_u \text{ and } \bar{N}_u \]

\[ h = \frac{-k_f \frac{\partial T}{\partial y}}{T_s - T_{\infty}} \]
\[ N_u = \frac{h(x) \cdot \frac{x}{k_f}}{k_f} \quad \text{(different w/ Bi!)} \]
\[ \bar{h} = \frac{1}{A_s} \int h \, dA_s \]
\[ \bar{N}_u = \frac{\bar{h} L}{k_f} \]

(Flat plate \( \bar{h} = \frac{1}{L} \int h \, dx \))
(1) Solve for temperature profile (6.4-6.5)

Continuity eqn. \( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \) (incompressible fluid)

\( x \)-momentum eqn. \( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \)

(constant properties negligible body forces)

\( y \)-momentum eqn \( \frac{\partial p}{\partial y} = 0 \)

Energy eqn. \( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{\rho} (\frac{\partial u}{\partial y})^2 \)

\( i = 0 \)

(2) Boundary Layer Similarity (Table 6.1 & 6.3)

a) \( Pr = \frac{\nu}{\alpha} \leq 1 \)

\( U^* = f_1(x^*, y^*, Re) \sim T^* = f_2(x^*, y^*, Re, Pr) \)

\( Re \)nold Analogy \( \frac{c_f}{2} = St = \frac{Nu}{Re Pr} \)

\( c_f = \frac{C_s}{\rho u_0^{1.2}} \)

b) \( 0.6 < Pr < 60 \)

Chilton-Colburn Analogy \( \frac{c_f}{2} = St \cdot Pr^{1/3} = f \)

(3) Correlations!

For a particular surface geometry

\( h = \overline{Nu} = f (x^*, Re, Pr) \)

\( \overline{h} = \overline{Nu} = + (Re, Pr) \)
Flow over flat plate:

\[
\delta = 5 \times Re_x^{-1/2} \\
C_f = 0.664 \times Re_x^{-1/2} \\
\overline{C_f} = 1.328 \times Re^{-1/2}
\]

\[
\overline{\delta_t} = \delta \cdot Pr^{-1/3} = 5 \times Re^{-1/2} \cdot Pr^{-1/3}
\]

\[
Nu_x = 0.322 \cdot Re_x^{1/2} \cdot Pr^{1/3} \quad (0.6 \leq Pr \leq 50)
\]

\[
Nu_x = 0.664 \cdot Re_x^{1/2} \cdot Pr^{1/3} \quad (0.6 \leq Pr \leq 50)
\]

Table 7.9  P394  Summary of heat transfer correlations.
How to apply a convection correlation

For internal flow
use \( T_m = \frac{T_{min} + T_{max}}{2} \)
or according to
the correlations

decide it is
fully developed/entrance
region

\[ T_f = \frac{T_s + T_w}{2} \]

specify reference temp.
evaluate fluid properties at that temp.

Calculate the Re number
Laminar or turbulent?

Decide \( N_u, \overline{N}_u \)

Select the appropriate correlation
Internal flow

Mean velocity
\[ \bar{u} = \frac{m}{PA_c} = \frac{\int p u dA_c}{PA_c} \]

For a circular tube
\[ \bar{u} = \frac{2}{r_0} \int_0^{r_0} u(r, x) r dr \]

What is fully developed region?

\[ \left. \frac{\partial \bar{u}}{\partial x} \right|_{fd,h} = 0 \]

Mean temperature
\[ T_m = \frac{\int p u C_v T dA_c}{m C_v} \]

For a circular tube, \( C_v, \rho = \text{const.} \)
\[ T_m = \frac{2}{Um r_0^2} \int_0^{r_0} u Tr \, dr \]

How \( T_m \) varies with \( x \)?

(8.3.1) General considerations
(8.3.2) Const. surface heat flux, \( q_s \)
\[ T_m(x) = T_{m,i} + \frac{q_s}{mC_p} x \]
(8.3.3) Const. surface temp., \( T_s \)
\[ \frac{\Delta T}{\Delta T_i} = \frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{mC_p h}\right) \]
Is the flow laminar or turbulent?

\[ \text{Re}_D = \frac{\rho u_m D}{\mu} = \frac{4 m}{\pi D m} \]

\[ \text{Re}_D, c \approx 2300 \]

Laminar flow:
\[ \frac{X_{turb}}{D} \approx 0.05 \text{Re}_D \text{Pr} \]

Turbulent flow:
\[ \frac{X_{turb}}{D} = 10 \]

10 ≤ \[ \frac{X_{turb}}{D} \leq 60 \]

Apply a correlation

Table 8.4 !

(8.4) Laminar flow in circular tube

1. Fully developed region (both hydrodynamic and thermal)

\[ \text{Nu}_D = \frac{h_D}{k} = 4.36 \]

\[ T_s = \text{const.} \]

\[ \text{Nu}_D = 3.66 \]

2. Entry Region (8.4.2)

\[ \log \text{Nu}_D \]

4.36

3.66

\[ \log \left( \frac{X}{\text{Re}_D \text{ Pr}} \right) = \text{log} (GZ^2) \]