iLab Homework Problem Solution

Please refer to the spreadsheet for all numerical answers based on a representative set of experimental data.

Part A: Countercurrent Flow

1.) Energy Balance: Assuming perfectly insulated heat exchanger

\[
\dot{m}_c \, c_{p,c} (T_{c,o} - T_{c,i}) = \dot{m}_h \, c_{p,h} (T_{h,i} - T_{h,o})
\]

where,

\[
\dot{m}_c = \frac{Q_c (\frac{1 \text{ m}^3}{1000 \text{ L}}) (\frac{1 \text{ min}}{60 \text{ s}}) \rho_c}{c_{p,c}} \quad \text{and} \quad \dot{m}_h = \frac{Q_h (\frac{1 \text{ m}^3}{1000 \text{ L}}) (\frac{1 \text{ min}}{60 \text{ s}}) \rho_h}{c_{p,h}}
\]

The rate of energy lost by the hot stream is equal to the right half of the energy balance shown above while the energy gained by the cold stream is the expression on the left.

\[
q_c = \dot{m}_c \, c_{p,c} (T_{c,o} - T_{c,i}) \quad q_h = \dot{m}_h \, c_{p,h} (T_{h,i} - T_{h,o})
\]

Ideally, \(q_c\) should equal \(q_h\). However, in real life, this can be very hard to achieve. It is unlikely that the rates would be equal in this experiment, but they should be relatively close. The difference could be due to heat loss to the environment (could be included as an additional term in the energy balance) since perfect insulation only exists in theory. Also, the manner in which the data was recorded was not very accurate so it is very likely that your average temperatures and flowrates are inaccurate and will mess up the results.

2.) Relating rate of heat transfer to overall heat transfer coefficient

\[
q = UAF \Delta T_{\text{lmCF}} = \frac{q_h + q_c}{2}
\]

The log mean temperature difference for pure countercurrent flow:

\[
\Delta T_{\text{lmCF}} = \frac{\Delta T_1 - \Delta T_2}{\ln \left( \frac{\Delta T_1}{\Delta T_2} \right)} \quad \Delta T_1 = T_{h,i} - T_{c,o} \quad \Delta T_2 = T_{h,o} - T_{c,i}
\]
Since this plate heat exchanger is not in perfect countercurrent flow, the log mean temperature difference is adjusted by F which can be read off Fig. 7 (handout) after calculating the appropriate P and R as defined in the handout.

\[ \begin{align*}
U &= \frac{q}{AF\Delta T_{lm}CF}, \text{ get F from Fig. 7 in the handout} \\
\end{align*} \]

The number of heat transfer units (NTU) can also be read off of the top plot of Figure 7 using the same P and R as you used to get F.

3.) Average individual heat transfer coefficient assuming negligible fouling

\[ \begin{align*}
\frac{1}{U} &= \frac{1}{\bar{h}} + \frac{t_{plates}}{k_{plates}} + \frac{1}{\bar{h}} \\
\end{align*} \]

solving for \( \bar{h} \),

\[ \bar{h} = \frac{2}{\frac{1}{U} - \frac{t_{plates}}{k_{plates}}} \]

4.) Plot your results and compare with:

\[ \log\left(\frac{\bar{Nu}}{Pr^{1/3}}\right) = \log(A) + m \log(Re) \]

So the slope of your plot is \( m \) and the intercept is \( \log(A) \). However, you should never represent data on a plot in the manner shown above. You can use whatever method you choose to get \( A \) and \( m \), but you plot must be \( \frac{\bar{Nu}}{Pr^{1/3}} \) vs. Re (not \( \log\left(\frac{\bar{Nu}}{Pr^{1/3}}\right) \) vs \( \log(Re) \)) with the numbers spaced according to a logarithmic scale.

It is likely that values of \( A \) and \( m \) will differ slightly from those given in Eqn. 8.59 in the text. For a given Re,

\[ \bar{Nu} = A \cdot Re^m \cdot Pr^{1/3} \]

It is very likely that your measurement is somewhat different than that predicted by the book. The correlation for turbulent flow in tubes should be similar to that for this plate heat exchanger. However, correlations are never perfect even when they are for the specific case in which you are dealing with. Therefore, some variation is expected. Most
of the difference probably results from poor recording of the data as mentioned previously.

5.) Calculate NTU

$$NTU_{\text{min}} = \frac{UA}{(mCp)_{\text{min}}}$$

Using R calculated before with NTU and Figure 6, get P. From P get $T_{c,o}$ and then get $T_{h,o}$ from energy balance, where.

$$T_{h,o} = T_{h,i} - \frac{m_c c_{p,c} (T_{c,o} - T_{c,i})}{m_h c_{p,h}}$$
### Counter Current Flow

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<tr>
<th>Flowrate (L/min)</th>
<th>0.5</th>
<th>0.5</th>
<th>0.5</th>
<th>0.5</th>
<th>.5 AVG</th>
<th>1.8</th>
<th>1.8</th>
<th>1.8</th>
<th>1.8</th>
<th>1.8 AVG</th>
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<tbody>
<tr>
<td>Thi (°C)</td>
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<td>38.69</td>
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<td>0.47</td>
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### Cocurrent Flow

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<td>Qc (L/min)</td>
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<td>5.00E-04</td>
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<th>Tave (K)</th>
<th>cp (kJ/kgK)</th>
<th>ν (m^3/kg)</th>
<th>A(m^2)</th>
<th>kplates (W/mK)</th>
<th>dplates (m)</th>
<th>viscosity (Ns/m^2)</th>
<th>kwater (W/mK)</th>
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<td>5.00E-04</td>
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</tr>
</tbody>
</table>
Part A

Problem 1

Flowrate (L/min) 0.5 1 1.8
\( \dot{m}_h \) (kg/s) 8.69E-03 1.67E-02 3.09E-02
\( \dot{m}_c \) (kg/s) 7.81E-03 1.58E-02 3.04E-02
\( q_h \) (W) 467.27 893.81 1523.09
\( q_c \) (W) 475.77 931.22 1505.12

Problem 2

Flowrate (L/min) 0.5 1 1.8
\( \Delta T_1 \) (K) 5.510 7.246 8.248
\( \Delta T_2 \) (K) 7.212 8.576 8.304
LMTD,\( \Delta F \) (K) 6.323 7.892 8.276
R 0.726 0.661 0.589
F (Fig 7) 0.72 0.82 0.91
U (W/m²K) 1438.54 1958.34 2792.31
NTU 4 2.2 1.7

Problem 3

Flowrate (L/min) 0.5 1 1.8
indiv. ave. HT coeff. (W/m²K) 3040.27 4225.44 6234.16

Problem 4

Flowrate (L/min) 0.5 1 1.8
\( \Pr \) 5.52 5.51 5.66
\( \Nu \) 14.79 20.55 30.41
V (m/s) 0.07 0.15 0.28
Re 273.91 540.40 994.30
\( \log(\Nu/\Pr^{1/3}) \) 0.9228631 1.0656687 1.2319901
\( \log(\Re) \) 2.44 2.73 3.00
A 0.3748867
\( \m \) 0.55
\( \Nu(\Re=1000) \) my correlation 29.589474
\( \Nu(\Re=1000) \) eqn 8.59 10.208574

Part B

Problem 5

\( \text{NTU} \) 2.14
\( R \) 0.944
P (Fig 6) 0.52
E (=P) 0.52
\( T_{c,o} \) (C) 27.64
\( T_{h,o} \) (C) 26.58