PROBLEM 3.99

KNOWN: Net radiative flux to absorber plate.

FIND: (a) Maximum absorber plate temperature, (b) Rate of energy collected per tube.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional (x) conduction along absorber plate, (3) Uniform radiation absorption at plate surface, (4) Negligible losses by conduction through insulation, (5) Negligible losses by convection at absorber plate surface, (6) Temperature of absorber plate at x = 0 is approximately that of the water.

PROPERTIES: Table A-1, Aluminum alloy (2024-T6): \( k = 180 \text{ W/m-K} \).

ANALYSIS: The absorber plate acts as an extended surface (a conduction-radiation system), and a differential equation which governs its temperature distribution may be obtained by applying Eq.1.11a to a differential control volume. For a unit length of tube

\[
q_x' + q_{rad} (dx) - q_{x+dx} = 0.
\]

With

\[
q_{x+dx} = q_x' + \frac{dq_x'}{dx} dx
\]

and

\[
q_x' = -kt \frac{dT}{dx}
\]

it follows that,

\[
q_{rad} = \frac{dT}{dx} \left[ -kt \frac{dT}{dx} \right] = 0
\]

\[
\frac{dT}{dx} + \frac{q_{rad}}{kt} = 0
\]

Integrating twice it follows that, the general solution for the temperature distribution has the form,

\[
T(x) = -\frac{q_{rad}}{2kt} x^2 + C_1 x + C_2.
\]

Continued …..
PROBLEM 3.99 (Cont.)

The boundary conditions are:

\[
\begin{align*}
T(0) &= T_w \\
\frac{dT}{dx} \bigg|_{x=L/2} &= 0 \\
C_2 &= T_w \\
C_1 &= \frac{q_{rad} L}{2kt}
\end{align*}
\]

Hence,

\[
T(x) = \frac{q_{rad}}{2kt} x (L-x) + T_w.
\]

The maximum absorber plate temperature, which is at \(x = L/2\), is therefore

\[
T_{\text{max}} = T(L/2) = \frac{q_{rad} L^2}{8kt} + T_w.
\]

The rate of energy collection per tube may be obtained by applying Fourier’s law at \(x = 0\). That is, energy is transferred to the tubes via conduction through the absorber plate. Hence,

\[
q' = 2 \left[ -k \frac{dT}{dx} \right]_{x=0}
\]

where the factor of two arises due to heat transfer from both sides of the tube. Hence,

\[
q' = -Lq_{rad}.
\]

Hence

\[
T_{\text{max}} = \frac{800 \text{ W/m}^2 (0.2 \text{ m})^2}{8 \left[ 180 \text{ W/m} \cdot \text{K} \right] (0.006 \text{ m})} + 60^\circ \text{C}
\]

or

\[
T_{\text{max}} = 63.7^\circ \text{C}
\]

and

\[
q' = -0.2 \text{ m} \times 800 \text{ W/m}^2
\]

or

\[
q' = -160 \text{ W/m}.
\]

COMMENTS: Convection losses in the typical flat plate collector, which is not evacuated, would reduce the value of \(q'\).
PROBLEM 3.120

KNOWN: Length, diameter, base temperature and environmental conditions associated with a brass rod.

FIND: Temperature at specified distances along the rod.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient $h$.

PROPERTIES: Table A-1, Brass ($T = 110^\circ$C): $k = 133$ W/m·K.

ANALYSIS: Evaluate first the fin parameter

$$m = \left[ \frac{hP}{kA} \right]^{1/2} = \left[ \frac{h\pi D}{k\pi D^2 / 4} \right]^{1/2} = \left[ \frac{4h}{kD} \right]^{1/2} = \left[ \frac{4 \times 30 \text{ W/m}^2 \cdot \text{K}}{133 \text{ W/m} \cdot \text{K} \times 0.005\text{m}} \right]^{1/2}$$

$$m = 13.43 \text{ m}^{-1}.$$

Hence, $mL = (13.43) \times 0.1 = 1.34$ and from the results of Example 3.8, it is advisable not to make the infinite rod approximation. Thus from Table 3.4, the temperature distribution has the form

$$\theta = \frac{\cosh mL + (h/mk)\sinh mL}{\cosh mL + (h/mk)\sinh mL} \theta_b$$

Evaluating the hyperbolic functions, $\cosh mL = 2.04$ and $\sinh mL = 1.78$, and the parameter

$$\frac{h}{mk} = \frac{30 \text{ W/m}^2 \cdot \text{K}}{133 \text{ W/m} \cdot \text{K}} = 0.0168,$$

with $\theta_b = 180^\circ$C the temperature distribution has the form

$$\theta = \frac{\cosh mL - 0.0168 \sinh mL}{2.07} \left(180^\circ\text{C}\right).$$

The temperatures at the prescribed location are tabulated below.

<table>
<thead>
<tr>
<th>$x$ (m)</th>
<th>$\cosh mL-x$</th>
<th>$\sinh mL-x$</th>
<th>$\theta$ (°C)</th>
<th>$T$ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1 = 0.025$</td>
<td>1.55</td>
<td>1.19</td>
<td>136.5</td>
<td>156.5</td>
</tr>
<tr>
<td>$x_2 = 0.05$</td>
<td>1.24</td>
<td>0.725</td>
<td>108.9</td>
<td>128.9</td>
</tr>
<tr>
<td>$L = 0.10$</td>
<td>1.00</td>
<td>0.00</td>
<td>87.0</td>
<td>107.0</td>
</tr>
</tbody>
</table>

COMMENTS: If the rod were approximated as infinitely long: $T(x_1) = 148.7^\circ$C, $T(x_2) = 112.0^\circ$C, and $T(L) = 67.0^\circ$C. The assumption would therefore result in significant underestimates of the rod temperature.
PROBLEM 4.19

KNOWN: Tube embedded in the center plane of a concrete slab.

FIND: (a) The shape factor and heat transfer rate per unit length using the appropriate tabulated relation, (b) Shape factor using flux plot method.

SCHEMATIC:

ASSUMPTIONS: (1) Two-dimensional conduction, (2) Steady-state conditions, (3) Constant properties, (4) Concrete slab infinitely long in horizontal plane, $L >> z$.

PROPERTIES: Table A-3, Concrete, stone mix (300K): $k = 1.4 \text{ W/m}\cdot\text{K}$.

ANALYSIS: (a) If we relax the restriction that $z >> D/2$, the embedded tube-slab system corresponds to the fifth case of Table 4.1. Hence,

$$S = \frac{2\pi L}{\ell \ln (8z/\pi D)}$$

where $L$ is the length of the system normal to the page, $z$ is the half-thickness of the slab and $D$ is the diameter of the tube. Substituting numerical values, find

$$S = 6.72 L.$$

Hence, the heat rate per unit length is

$$q' = \frac{q}{L} = \frac{S}{L} k (T_1 - T_2) = 6.72 \times 1.4 \frac{W}{m \cdot K} (85 - 20) ^\circ C = 612 \text{ W}.$$

(b) To find the shape factor using the flux plot method, first identify the symmetrical section bounded by the symmetry adiabats formed by the horizontal and vertical center lines. Selecting four temperature increments ($N = 4$), the flux plot can then be constructed.

From Eq. 4.26, the shape factor of the symmetrical section is

$$S_0 = ML/N = 6L/4 = 1.5L.$$

For the tube-slab system, it follows that $S = 4S_0 = 6.0L$, which compares favorably with the result obtained from the shape factor relation.
**PROBLEM 4.67**

**KNOWN:** Long rectangular bar having one boundary exposed to a convection process \((T_\infty, h)\) while the other boundaries are maintained at constant temperature \(T_s\).

**FIND:** Using the finite-element method of FEHT, (a) Determine the temperature distribution, plot the isotherms, and identify significant features of the distribution, (b) Calculate the heat rate per unit length \((W/m)\) into the bar from the air stream, and (c) Explore the effect on the heat rate of increasing the convection coefficient by factors of two and three; explain why the change in the heat rate is not proportional to the change in the convection coefficient.

**SCHEMATIC:**

**ASSUMPTIONS:** (1) Steady-state, two dimensional conduction, (2) Constant properties.

**ANALYSIS:** (a) The symmetrical section shown in the schematic is drawn in FEHT with the specified boundary conditions and material property. The View | Temperature Contours command is used to represent ten isotherms (isopotentials) that have minimum and maximum values of 53.9°C and 85.9°C, respectively.

Because of the symmetry boundary condition, the isotherms are normal to the center-plane indicating an adiabatic surface. Note that the temperature change along the upper surface of the bar is substantial (= 40°C), whereas the lower half of the bar has less than a 3°C change. That is, the lower half of the bar is largely unaffected by the heat transfer conditions at the upper surface.

(b, c) Using the View | Heat Flows command considering the upper surface boundary with selected convection coefficients, the heat rates into the bar from the air stream were calculated.

\[
\begin{align*}
h (W/m^2 \cdot K) & \quad 100 \quad 200 \quad 300 \\
q' (W/m) & \quad 128 \quad 175 \quad 206
\end{align*}
\]

Increasing the convection coefficient by factors of 2 and 3, increases the heat rate by 37% and 61%, respectively. The heat rate from the bar to the air stream is controlled by the thermal resistances of the bar (conduction) and the convection process. Since the conduction resistance is significant, we should not expect the heat rate to change proportionally to the change in convection resistance.
PROBLEM 5.4

KNOWN: Plate initially at a uniform temperature $T_i$ is suddenly subjected to convection process $(T_\infty, h)$ on both surfaces. After elapsed time $t_0$, plate is insulated on both surfaces.

FIND: (a) Assuming $Bi \gg 1$, sketch on $T$ - $x$ coordinates: initial and steady-state ($t \to \infty$) temperature distributions, $T(x,t_0)$ and distributions for two intermediate times $t_0 < t < \infty$, (b) Sketch on $T$ - $t$ coordinates midplane and surface temperature histories, (c) Repeat parts (a) and (b) assuming $Bi \ll 1$, and (d) Obtain expression for $T(x,\infty) = T_f$ in terms of plate parameters $(M,c_p)$, thermal conditions $(T_i, T_\infty, h)$, surface temperature $T(L,t)$ and heating time $t_0$.

SCHEMATIC:

ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal generation, (4) Plate is perfectly insulated for $t > t_0$, (5) $T(0, t < t_0) < T_\infty$.

ANALYSIS: (a,b) With $Bi \gg 1$, appreciable temperature gradients exist in the plate following exposure to the heating process.

On $T$-$x$ coordinates: (1) initial, uniform temperature, (2) steady-state conditions when $t \to \infty$, (3) distribution at $t_0$ just before plate is covered with insulation, (4) gradients are always zero (symmetry), and (5) when $t > t_0$ (dashed lines) gradients approach zero everywhere.

(c) If $Bi \ll 1$, plate is space-wise isothermal (no gradients). On $T$-$t$ coordinates, the temperature distributions are flat; on $T$-$t$ coordinates, $T(L,t) = T(0,t)$.

(d) The conservation of energy requirement for the interval of time $\Delta t = t_0$ is

$$E_{in} - E_{out} = \Delta E = E_{final} - E_{initial} \quad 2 \int_0^{t_0} h A_s \left[ T_\infty - T(L,t) \right] dt = 0 = Mc_p \left( T_f - T_i \right)$$

where $E_{in}$ is due to convection heating over the period of time $t = 0 \to t_0$. With knowledge of $T(L,t)$, this expression can be integrated and a value for $T_f$ determined.
PROBLEM 5.6

KNOWN: The temperature-time history of a pure copper sphere in an air stream.

FIND: The heat transfer coefficient between the sphere and the air stream.

SCHEMATIC:

\[ T_\infty = 27°C \]
\[ T(0) = 66°C \]
\[ T(69s) = 55°C \]
\[ D = 12.7mm \]

ASSUMPTIONS: (1) Temperature of sphere is spatially uniform, (2) Negligible radiation exchange, (3) Constant properties.

PROPERTIES: Table A-1, Pure copper (333K): \( \rho = 8933 \text{ kg/m}^3 \), \( c_p = 389 \text{ J/kg K} \), \( k = 398 \text{ W/m K} \).

ANALYSIS: The time-temperature history is given by Eq. 5.6 with Eq. 5.7.

\[
\frac{\theta (t)}{\theta_i} = \exp \left( -\frac{t}{R_tC_t} \right) \quad \text{where} \quad R_t = \frac{1}{hA_s} \quad A_s = \pi D^2 \quad C_t = \rho Vc_p \quad V = \frac{\pi D^3}{6} \quad \theta = T - T_\infty.
\]

Recognize that when \( t = 69s \),

\[
\frac{\theta (t)}{\theta_i} = \frac{(55 - 27)°C}{(66 - 27)°C} = 0.718 = \exp \left( -\frac{t}{\tau_t} \right) = \exp \left( -\frac{69s}{\tau_t} \right)
\]

and noting that \( \tau_t = R_tC_t \) find

\( \tau_t = 208s. \)

Hence,

\[
h = \frac{\rho Vc_p}{A_s \tau_t} = \frac{8933 \text{ kg/m}^3 (\pi 0.0127^3 \text{ m}^3 / 6) 389 \text{ J/kg K}}{\pi 0.0127^2 \text{ m}^2 \times 208s} = 35.3 \text{ W/m}^2 \cdot \text{K}.
\]

COMMENTS: Note that with \( L_c = D_o/6 \),

\[
Bi = \frac{hL_c}{k} = 35.3 \text{ W/m}^2 \cdot \text{K} \times \frac{0.0127}{6} \text{ m/398 W/m} \cdot \text{K} = 1.88 \times 10^{-4}.
\]

Hence, \( Bi < 0.1 \) and the spatially isothermal assumption is reasonable.