PROBLEM 12.004

KNOWN: Furnace with prescribed aperture and emissive power.

FIND: (a) Position of gauge such that irradiation is \( G = 1000 \text{ W/m}^2 \), (b) Irradiation when gauge is tilted \( \theta_d = 20^\circ \), and (c) Compute and plot the gage irradiation, \( G \), as a function of the separation distance, \( L \), for the range \( 100 \leq L \leq 300 \text{ mm} \) and tilt angles of \( \theta_d = 0, 20, \) and \( 60^\circ \).

SCHEMATIC:

ASSUMPTIONS: (1) Furnace aperture emits diffusely, (2) \( A_d \ll L^2 \).

ANALYSIS: (a) The irradiation on the detector area is defined as the power incident on the surface per unit area of the surface. That is

\[
G = q_{f \rightarrow d} / A_d
\]

where \( q_{f \rightarrow d} \) is the radiant power which leaves \( A_f \) and is intercepted by \( A_d \). From Eqs. 12.2 and 12.5, \( \omega_{d-f} \) is the solid angle subtended by surface \( A_d \) with respect to \( A_f \),

\[
\omega_{d-f} = A_d \cos \theta_d / L^2 .
\]

Noting that since the aperture emits diffusely, \( I_e = E / \pi \) (see Eq. 12.14), and hence

\[
G = (E / \pi) A_f \cos \theta_f \left(A_d \cos \theta_d / L^2 \right) / A_d
\]

Solving for \( L^2 \) and substituting for the condition \( \theta_f = 0^\circ \) and \( \theta_d = 0^\circ \),

\[
L^2 = E \cos \theta_f \cos \theta_d A_f / \pi G .
\]

(b) When \( \theta_d = 20^\circ \), \( q_{f \rightarrow d} \) will be reduced by a factor of \( \cos \theta_d \) since \( \omega_{d-f} \) is reduced by a factor \( \cos \theta_d \). Hence,

\[
G = 1000 \text{ W/m}^2 \times \cos \theta_d = 1000 \times 0.94 = 940 \text{ W/m}^2.
\]

(c) Using the IHT workspace with Eq. (4), \( G \) is computed and plotted as a function of \( L \) for selected \( \theta_d \). Note that \( G \) decreases inversely as \( L^2 \). As expected, \( G \) decreases with increasing \( \theta_d \) and in the limit, approaches zero as \( \theta_d \) approaches \( 90^\circ \).
PROBLEM 12.17

KNOWN: Sun has equivalent blackbody temperature of 5800 K. Diameters of sun and earth as well as separation distance are prescribed.

FIND: Temperature of the earth assuming the earth is black.

SCHEMATIC:

ASSUMPTIONS: (1) Sun and earth emit as blackbodies, (2) No attenuation of solar irradiation enroute to earth, and (3) Earth atmosphere has no effect on earth energy balance.

ANALYSIS: Performing an energy balance on the earth,

\[ \dot{E}_{in} - \dot{E}_{out} = 0 \]

\[ A_{e,p} \cdot G_S = A_{e,s} \cdot E_b \left( T_e \right) \]

\[ \left( \pi D_e^2 / 4 \right) G_S = \pi D_e^2 \sigma T_e^4 \]

\[ T_e = \left( G_S / 4\sigma \right)^{1/4} \]

where \( A_{e,p} \) and \( A_{e,s} \) are the projected area and total surface area of the earth, respectively. To determine the irradiation \( G_S \) at the earth’s surface, equate the rate of emission from the sun to the rate at which this radiation passes through a spherical surface of radius \( R_{S,e} - D_e/2 \).

\[ \dot{E}_{in} - \dot{E}_{out} = 0 \]

\[ \pi D_S^2 \sigma T_S^4 = 4\pi \left[ R_{S,e} - D_e / 2 \right]^2 G_S \]

\[ \pi \left( 1.39 \times 10^9 \text{ m} \right)^2 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \cdot (5800 \text{ K})^4 \]

\[ = 4\pi \left[ 1.5 \times 10^{11} - 1.29 \times 10^7 / 2 \right]^2 \text{ m}^2 \times G_S \]

\[ G_S = 1377.5 \text{ W/m}^2. \]

Substituting numerical values, find

\[ T_e = \left( 1377.5 \text{ W/m}^2 / 4 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \right)^{1/4} = 279 \text{ K.} \]

COMMENTS: (1) The average earth’s temperature is greater than 279 K since the effect of the atmosphere is to reduce the heat loss by radiation.

(2) Note carefully the different areas used in the earth energy balance. Emission occurs from the total spherical area, while solar irradiation is absorbed by the projected spherical area.
PROBLEM 12.58

KNOWN: Spectral transmissivity of low iron glass (see Fig. 12.24).

FIND: Interpretation of the greenhouse effect.

SCHEMATIC:

![Diagram of a greenhouse with low-iron glass and interior temperature labeled]

ANALYSIS: The glass affects the net radiation transfer to the contents of the greenhouse. Since most of the solar radiation is in the spectral region $\lambda < 3 \text{ \mu m}$, the glass will transmit a large fraction of this radiation. However, the contents of the greenhouse, being at a comparatively low temperature, emit most of their radiation in the medium to far infrared. This radiation is not transmitted by the glass. Hence the glass allows short wavelength solar radiation to enter the greenhouse, but does not permit long wavelength radiation to leave.
**PROBLEM 12.71**

**KNOWN:** Temperature, absorptivity, transmissivity, radiosity and convection conditions for a semitransparent plate.

**FIND:** Plate irradiation and total hemispherical emissivity.

**SCHEMATIC:**

**ASSUMPTIONS:** (1) Steady-state conditions, (2) Uniform surface conditions.

**ANALYSIS:** From an energy balance on the plate

\[ \dot{E}_{in} = \dot{E}_{out} \]

\[ 2G = 2q_{\text{conv}} + 2J. \]

Solving for the irradiation and substituting numerical values,

\[ G = 40 \text{ W/m}^2 \cdot K (350 - 300) \text{ K} + 5000 \text{ W/m}^2 = 7000 \text{ W/m}^2. \]

From the definition of J,

\[ J = E + \rho G + \tau G = E + (1 - \alpha) G. \]

Solving for the emissivity and substituting numerical values,

\[ \varepsilon = \frac{J - (1 - \alpha) G}{\sigma T^4} = \left( \frac{5000 \text{ W/m}^2}{5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4} \cdot (350 \text{ K})^4 \right) = 0.94. \]

Hence,

\[ \alpha \neq \varepsilon \]

and the surface is not gray for the prescribed conditions.

**COMMENTS:** The emissivity may also be determined by expressing the plate energy balance as

\[ 2\alpha G = 2q_{\text{conv}} + 2E. \]

Hence

\[ \varepsilon \sigma T^4 = \alpha G - h(T - T_\infty) \]

\[ \varepsilon = \frac{0.4 \left( 7000 \text{ W/m}^2 \right) - 40 \text{ W/m}^2 \cdot K (50 \text{ K})}{5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (350 \text{ K})^4} = 0.94. \]
PROBLEM 13.1

KNOWN: Various geometric shapes involving two areas $A_1$ and $A_2$.

FIND: Shape factors, $F_{12}$ and $F_{21}$, for each configuration.

ASSUMPTIONS: Surfaces are diffuse.

ANALYSIS: The analysis is not to make use of tables or charts. The approach involves use of the 
reciprocity relation, Eq. 13.3, and summation rule, Eq. 13.4. Recognize that reciprocity applies to two 
surfaces; summation applies to an enclosure. Certain shape factors will be identified by inspection. 
Note L is the length normal to page.

(a) Long duct (L):

![Diagram of a long duct with $A_1$ and $A_2$] 

By inspection, $F_{12} = 1.0$  

By reciprocity, 

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2RL}{(3/4) \cdot 2\pi RL} \times 1.0 = \frac{4}{3\pi} = 0.424$$

(b) Small sphere, $A_1$, under concentric hemisphere, $A_2$, where $A_2 = 2A$

Summation rule 

$$F_{11} + F_{12} + F_{13} = 1$$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$  

By reciprocity, 

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{A_1}{2A_1} \times 0.5 = 0.25.$$

(c) Long duct (L):

![Diagram of a long duct with $A_1$ and $A_2$] 

By inspection, $F_{12} = 1.0$  

By reciprocity, 

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2RL}{\pi RL} \times 1.0 = \frac{2}{\pi} = 0.637$$

(d) Long inclined plates (L):

![Diagram of long inclined plates] 

Summation rule 

$$F_{11} + F_{12} + F_{13} = 1$$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$  

By reciprocity, 

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{20L}{10(2)^{1/2} L} \times 0.5 = 0.707.$$

(e) Sphere lying on infinite plane

![Diagram of a sphere on an infinite plane] 

Summation rule, 

$$F_{11} + F_{12} + F_{13} = 1$$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.5$  

By reciprocity, 

$$F_{21} = \frac{A_1}{A_2} F_{12} \rightarrow 0 \text{ since } A_2 \rightarrow \infty.$$
PROBLEM 13.1 (Cont.)

(f) Hemisphere over a disc of diameter D/2; find also F_{22} and F_{23}.

By inspection, F_{12} = 1.0

Summation rule for surface A_3 is written as

F_{31} + F_{32} + F_{33} = 1. Hence, F_{32} = 1.0.

By reciprocity,

\[ F_{23} = \frac{A_3}{A_2} F_{32} \]

\[ F_{23} = \left[ \frac{\pi D^2}{4} - \frac{\pi (D/2)^2}{4} \right] / \frac{\pi D^2}{2} = 1.0 \times 0.375 = 0.375. \]

By reciprocity,

\[ F_{21} = \frac{A_1}{A_2} F_{12} = \left( \frac{\pi D^2}{4} / \frac{\pi D^2}{2} \right) \times 1.0 = 0.125. \]

Summation rule for A_2,

F_{21} + F_{22} + F_{23} = 1 or

F_{22} = 1 - F_{21} - F_{23} = 1 - 0.125 - 0.375 = 0.5.

Note that by inspection you can deduce F_{22} = 0.5

(g) Long open channel (L):

Summation rule for A_1

F_{11} + F_{12} + F_{13} = 0

but F_{12} = F_{13} by symmetry, hence F_{12} = 0.50.

By reciprocity,

\[ F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2 \times L}{(2\pi)/4 \times L} = \frac{4}{\pi} \times 0.50 = 0.637. \]

COMMENTS: (1) Note that the summation rule is applied to an enclosure. To complete the enclosure, it was necessary in several cases to define a third surface which was shown by dashed lines.

(2) Recognize that the solutions follow a systematic procedure; in many instances it is possible to deduce a shape factor by inspection.
**PROBLEM 13.7**

**KNOWN:** Right-circular cylinder of diameter D, length L and the areas $A_1$, $A_2$, and $A_3$ representing the base, inner lateral and top surfaces, respectively.

**FIND:** (a) Show that the view factor between the base of the cylinder and the inner lateral surface has the form

$$F_{12} = 2H \left[ (1 + H^2)^{1/2} - H \right]$$

where $H = L/D$, and (b) Show that the view factor for the inner lateral surface to itself has the form

$$F_{22} = 1 + H - \left(1 + H^2\right)^{1/2}$$

**SCHEMATIC:**

**ASSUMPTIONS:** Diffuse surfaces with uniform radiosities.

**ANALYSIS:** (a) *Relation for $F_{12}$, base-to-inner lateral surface.* Apply the summation rule to $A_1$, noting that $F_{11} = 0$

$$F_{11} + F_{12} + F_{13} = 1$$

$$F_{12} = 1 - F_{13} \quad (1)$$

From Table 13.2, Fig. 13.5, with $i = 1$, $j = 3$,

$$F_{13} = \frac{1}{2} \left[ S - \left( S^2 - 4(D_3/D_1)^2 \right)^{1/2} \right] \quad (2)$$

$$S = 1 + \frac{1 + R_3^2}{R_1^2} = \frac{1}{R^2} + 2 = 4H^2 + 2 \quad (3)$$

where $R_1 = R_3 = R = D/2L$ and $H = L/D$. Combining Eqs. (2) and (3) with Eq. (1), find after some manipulation

Continued .....
PROBLEM 13.7 (Cont.)

\[
F_{12} = 1 - \frac{1}{2} \left\{ 4H^2 + 2 - \left( 4H^2 + 2 \right)^2 - 4 \right\}^{1/2}
\]

\[
F_{12} = 2H \left[ \left( 1 + H^2 \right)^{1/2} - H \right]
\]

(b) Relation for \(F_{22}, \text{inner lateral surface}\). Apply summation rule on \(A_2\), recognizing that \(F_{23} = F_{21}\),

\[
F_{21} + F_{22} + F_{23} = 1 \quad F_{22} = 1 - 2F_{21}
\]

Apply reciprocity between \(A_1\) and \(A_2\),

\[
F_{21} = \left( \frac{A_1}{A_2} \right) F_{12}
\]

and substituting into Eq. (5), and using area expressions

\[
F_{22} = 1 - 2 \frac{A_1}{A_2} F_{12} = 1 - 2 \frac{D}{4L} F_{12} = 1 - \frac{1}{2H} F_{12}
\]

where \(A_1 = \pi D^2/4\) and \(A_2 = \pi DL\).

Substituting from Eq. (4) for \(F_{12}\), find

\[
F_{22} = 1 - \frac{1}{2} \frac{H}{2} \left[ \left( 1 + H^2 \right)^{1/2} - H \right] = 1 + H - \left( 1 + H^2 \right)^{1/2}
\]

<
PROBLEM 13.19

KNOWN: Arrangement of three black surfaces with prescribed geometries and surface temperatures.

FIND: (a) View factor $F_{13}$, (b) Net radiation heat transfer from $A_1$ to $A_3$.

SCHEMATIC:

**ASSUMPTIONS:** (1) Interior surfaces behave as blackbodies, (2) $A_2 \gg A_1$.

**ANALYSIS:** (a) Define the enclosure as the interior of the cylindrical form and identify $A_4$.

Applying the view factor summation rule, Eq. 13.4,

$$F_{11} + F_{12} + F_{13} + F_{14} = 1.$$  \hfill (1)

Note that $F_{11} = 0$ and $F_{14} = 0$. From Eq. 13.8,

$$F_{12} = \frac{D^2}{D^2 + 4L^2} = \frac{(3m)^2}{(3m)^2 + 4(2m)^2} = 0.36.$$  \hfill (2)

From Eqs. (1) and (2),

$$F_{13} = 1 - F_{12} = 1 - 0.36 = 0.64.$$  \hfill <

(b) The net heat transfer rate from $A_1$ to $A_3$ follows from Eq. 13.13,

$$q_{13} = A_1 F_{13} \sigma (T_1^4 - T_3^4)$$

$$q_{13} = 0.05m^2 \times 0.64 \times 5.67 \times 10^{-8} W/m^2 \cdot K^4 \left(1000^4 - 500^4 \right) K^4 = 1700 W.$$  \hfill <

**COMMENTS:** Note that the summation rule, Eq. 13.4, applies to an enclosure; that is, the total region above the surface must be considered.