1. Raindrops falling in Boston may sometimes be contaminated owing to absorption of gaseous pollutants while falling through the air. The highest average daily SO$_2$ concentration in the air over Boston is around $0.36 \times 10^{-4}$ percent by volume. When this concentration exists uniformly throughout the air, what is the SO$_2$ concentration in a 0.1 cm raindrop reaching the ground after falling for 5 minutes in the polluted air?

The effect of the relative velocity between the drop and the air may be neglected; i.e., for diffusion purposes, it may be assumed that the drops are stationary in the air with $D_{SO_2-Air} = 0.2$ cm$^2$/sec.

Within the drop, $D_{SO_2-H_2O} = 2 \times 10^{-5}$ cm$^2$/sec. The temperature and pressure may be assumed uniform at 60ºF and 1 atm. Dilute solutions of SO$_2$ in H$_2$O obey Henry’s law, for example:

$$p_v = Hc$$

where $p_v$ is the vapor pressure of SO$_2$ in equilibrium with the solution in which the SO$_2$ concentration is $c$, and $H$ is constant. It is known that $H = 180$ cm$^3$atm/g-mole for the above conditions.
Many insects breathe through structures called spiracles, which are essentially open tubes extending deep into the insect’s body as shown below. Oxygen diffuses into these tubes from the surrounding air and gas exchange occurs along the walls. It can be assumed that about one mole of carbon dioxide diffuses into the spiracle gas phase for every mole of oxygen that diffuses out of the spiracle into the tissue. To prevent excessive water loss, the walls of the spiracles are coated with a cuticle which serves as a diffusion barrier for gases. The thickness of this cuticle is 10 microns, except at the end (x = L), where it is thicker and the flux of oxygen is zero out the end of the tube. Outside the cuticle, in the tissue, the concentration of oxygen is essentially constant and is approximately 5% of the concentration which would be in equilibrium with air. Comparative entomologists (scientists who study insects and compare their features) are interested in understanding how the ability of various insects to exchange gases with the environment limits the size or activity level of the insects. For example, it has been proposed that larger insects are found in the tropics than in arctic regions because diffusion is faster in warm equatorial climates. You want to show the entomologists how to quantitatively assess their hypotheses by providing a mathematical model of insect respiration.

(a) What is the local flux of oxygen from the spiracle to the tissue at any point along the wall in terms of the gas phase concentration driving force and other relevant parameters? Note that the cuticle is thin relative to the radius of the spiracle.

(b) Use your answer from (a) to derive a differential equation for the oxygen concentration within the spiracle, $c_s$, as a function of $x$. Validate the assumption that radial concentration gradients are negligible in the gas phase.

(c) Does the oxygen concentration in the spiracle ever drop to less than 20% of ambient for the case of an insect in the tropics (data shown below)? (You do not need to show all the steps in the solution of your equation in (b) if you cite a relevant solution in your notes or text.)

(d) Is the spiracle an efficient way to exchange gases? The efficiency of the spiracle can be judged by comparing the amount of oxygen which is transferred within the spiracle to the amount which would be transferred if the ambient oxygen concentration prevailed in the gas phase all along the length of the spiracle.

Data (all at 35 °C, the temperature of the tropics):
spiral radius, $r = 0.01$ cm
spiral length, $L = 0.8$ cm
cuticle thickness, $t = 0.001$ cm
Oxygen solubility in cuticle or tissues Relative to air, $c_r$ or $c_t = 0.025 c_o$.

$D_{o, cuticle} = 2 \times 10^{-5}$ cm$^2$/s
$D_{o, air} = 0.2$ cm$^2$/s
oxygen concentration in cuticle, $c_t = 0.01$ mmol/L
oxygen concentration in air, $c_a = 8$ mmol/L
3. In the oxidation of many metals, an oxide film is formed on the surface of the metal. For oxidation to proceed, oxygen must diffuse through the oxide film to the surface of the metal. As oxidation proceeds, the diffusion path increases with time. Eventually, the oxidation becomes diffusion controlled and the dissolved oxygen concentration at the oxide-metal interface becomes essentially zero. If a pseudo-steady-state diffusion-controlled condition may be assumed, develop an expression that relates the depth of the oxide film to elapsed time, oxygen concentration at the free surface of the oxide film, and the diffusivity of oxygen through the oxide. You may assume there is no volume change associated with oxidation of the metal. Frequently, however, there is a substantial volumetric expansion associated with oxidation. Please comment on the practical ramifications of this expansion.

4. In a heat exchanger which is to be used to produce liquid propane by condensation of propane vapor, there is a concern about the role of non-condensible vapors (e.g. N₂ and Ar). In particular, there is concern that non-condensibles will concentrate at the vapor – liquid interface and thereby diminish the partial pressure of the propane. The general situation is depicted in the figure. It is agreed that this issue will be examined in a section of the exchanger where the condensing flux is expected to be 150 lbm / ft² hr. It is assumed that the propane contains only a little nitrogen and that, for the C₃H₈/N₂ system, the Prandtl number and the Schmidt number are equal. The density of the vapor is 0.20 lbm / ft³. Furthermore, the heat transfer coefficient from the bulk vapor to the vapor – liquid interface is estimated to be 60 Btu/hr ft² º F. The N₂ concentration in the bulk vapor will be denoted as C₀ and that at the liquid – vapor interface by Cₛ.

a) Estimate the mass transfer coefficient (in ft/hr) for exchange between the bulk vapor and the liquid – vapor interface.

b) Estimate the value of Cₛ/C₀
5. A wetted wall column is to be used to study the absorption of CO₂ into water. The column is supplied with solute-free water at the top and air containing 5 mole percent CO₂ is supplied at the bottom. The air flow rate is such that the change in gas composition may be neglected. The gas phase resistance to mass transfer is negligible. The situation is as depicted in Sketch A.

The velocity profile in the draining film is given by:

\[ u = \frac{1}{2} \frac{\rho gd^2}{\mu} \left[ 1 - \frac{y^2}{d^2} \right] \]

(See Sketch B)

It has been proposed that the absorption be analyzed by using “penetration” theory and setting

\[ u \equiv \frac{1}{2} \frac{\rho gd^2}{\mu} \]

for all values of y which lie within the penetration depth.

The anticipated range of d is 0.1 mm to 0.3 mm.

A. Is the penetration theory valid for this range of d? Justify.

B. It is permissible to neglect axial diffusion in this problem. Please justify this conclusion.

C. Assuming the penetration theory concept and the above approximation for the velocity profile to be valid, develop an expression for the mass transfer coefficient, as a function of x and other appropriate parameters.

D. What is the CO₂ flux (gm/cm² sec) at x = 10 cm if d = 0.3 mm?

See Sketches A & B next page.
Properties, etc.

\[ \begin{align*}
\rho_{\text{H}_2\text{O}} &= 1 \text{ gm/cm}^3 \\
\nu_{\text{H}_2\text{O}} &= 0.01 \text{ cm}^2/\text{sec} \\
g &= 980 \text{ cm/sec}^2 \\
D_{\text{CO}_2, \text{H}_2\text{O}} &= 2 \times 10^{-5} \text{ cm}^2/\text{sec} \\
0.1 \text{mm} < d < 0.3 \text{ mm} \\
\text{CO}_2 \text{ solubility} &= 0.15 \text{ gm CO}_2/100 \text{ gm H}_2\text{O} \\
\text{at a CO}_2 \text{ partial pressure of 1 atm.}
\end{align*} \]
6. A liquid/liquid extractor is being studied to determine the values of the “film coefficients” which characterize its performance. In the extractor, an aqueous solution forms the continuous phase and toluene drops which are dispersed by agitation constitute the organic phase. The volume ratio of the aqueous and organic phases is 2:1 (V_{aq}/V_{org} = 2). It is also known that the interfacial area for transport, a, is 20 cm²/cm³ of toluene. Furthermore, a “lumped” analysis may be employed in order to describe the device, i.e., it may be assumed that concentrations are uniform within each phase and all of the resistance to transport is associated with the interface.

Two separate experiments using quite different solutes, X and Y, have been performed in which the solute is dissolved in the aqueous phase and then extracted into the organic phase. In each case, the initial concentration in the aqueous phase was 0.01M.

The results are as follows:

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>Concentration of X in organic phase (mol/1)</th>
<th>Concentration of Y in organic phase (mol/1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>3.2 x 10^{-4}</td>
<td>7.9 x 10^{-5}</td>
</tr>
<tr>
<td>50</td>
<td>1.5 x 10^{-3}</td>
<td>3.8 x 10^{-4}</td>
</tr>
<tr>
<td>200</td>
<td>5.3 x 10^{-3}</td>
<td>1.3 x 10^{-3}</td>
</tr>
<tr>
<td>1000</td>
<td>1.4 x 10^{-2}</td>
<td>3.5 x 10^{-3}</td>
</tr>
</tbody>
</table>

It is believed that solutes X and Y have identical diffusivities in water. The diffusivities in toluene are different than in water, but still the same for X and Y. However, the partition coefficient for X and Y is quite different. It is defined here as:

\[
S_{ol/Aq} = \frac{\text{equilibrium concentration in toluene}}{\text{equilibrium concentration in water}}
\]

and it is known that \(S_{ol/Aq} = 8\) for X and \(S_{ol/Aq} = 0.5\) for Y.

a) What is the overall transport coefficient for solute X based on an overall organic phase driving force? Based on an overall aqueous phase driving force?

b) What is the overall transport coefficient for solute Y based on an overall organic phase driving force? Based on an overall aqueous phase driving force?

c) What are the values of the individual coefficients for the two phases?

d) Does one or the other phase pose a dominant resistance for the transport of solute X? If so, which phase? Repeat for solute Y.
Notes:
In the above, the individual coefficients are defined by:
\[ N^* = h_{m,aq}(C_{aq} - C_{aq,i}) = h_{m,o}(C_{o,i} - C_{o}) \]
Where “i” refers to the interface
“o” refers to the organic phase
and “aq” refers to the aqueous phase

Overall coefficients are defined by:
\[ N^* = U_{m,o}(S_{o/Aq}C_{aq} - C_{o}) = U_{m,aq}\left(C_{aq} - \frac{C_{o}}{S_{o/Aq}}\right) \]

7. Consider a single spherical ice crystal growing in a stagnant sea of subcooled salt water. The fluid is water at -4°C containing 0.04 gm NaCl/cm³; and all NaCl is excluded from the interior of the growing ice particle.

(a) If the diffusivity of sodium chloride in water were (for the moment) assumed to be infinite, how long would it take for the ice crystal to grow from infinitesimal size to a radius of 0.1 cm? You may assume the temperature field around the ice particle to be quasi-steady.

(b) Demonstrate whether the assumption of quasi-steady behavior is satisfactory.

(c) In truth, the diffusivity of sodium chloride in water is about 1.2 x 10⁻⁵ cm²/sec. Please develop a better estimate of the time for a crystal to grow to a radius of 0.1 cm. You may assume that both the temperature and the concentration fields are quasi-steady. Since salt is totally excluded from the growing crystal, note that the flux of salt at the surface of the crystal is simply \( C_i \frac{dR}{dt} \) where \( C_i \) is the salt concentration at the surface of the crystal and \( R \) is the radius of the sphere. This flux due to rejection must equal the quasi-steady rate at which salt diffuses away.

Data
- The freezing point depression is given by: \( \Delta T_{FPD} = 60 \cdot C_{NaCl} \)
  where \( \Delta T_{FPD} \) is in degrees Kelvin and \( C_{NaCl} \) is in gms/cm³.
- The heat of fusion for water is about 250J/gm
- The density of water and ice may be assumed equal at the value of 1 gm/cm³.
- Other physical properties of liquid water include the following:
  thermal conductivity = 5.7 x 10⁻³ W/cm K
  kinematic viscosity = 0.018 cm²/sec
  thermal diffusivity = 1.3 x 10⁻³ cm²/sec
  specific heat = 4.2 J/gm K
8. Consider a droplet of water with an initial diameter of 1 mm. The droplet is placed in slightly humid, stagnant air. You may assume that the water concentration is dilute, i.e. diffusion-induced convection in the air may be neglected.

a. If the air temperature is 25°C and the humidity of the air at large distances from the drop is 0.006 kg of water/kg of air, how much time will be required to totally evaporate the drop?

b. If the temperature of the air is sufficiently high, the dilute assumption fails and diffusion-induced convection must be included in the analysis. Will this convection tend to increase or decrease the heat flux? Will it tend to increase or decrease the mass flux? Will the temperature of the evaporating drop increase or decrease relative to the no convection prediction? Justify your choices.

c. In accordance with item (c), under the Data below, the process is quasi-steady, i.e. the drop temperature is independent of time. Please develop an expression which gives a rough estimate of the time required to establish this condition. Note that there are two time scales which may be much shorter than the evaporation time. One is the time required to establish the temperature profile in the gas phase. The other is the time required to cool (or heat) the droplet from its initial temperature to the evaporation temperature.

Data

(a) Water:
\[ \rho = 1000 \text{ kg/m}^3 \]
\[ c_p = 4200 \text{ J/kg} \cdot \text{K} \]
\[ \lambda_g = 2.4 \times 10^6 \text{ J/kg} \]
\[ k = 0.67 \text{ W/m} \cdot \text{k} \]

(b) Air-Water:
\[ D = 0.26 \times 10^{-4} \text{ m}^2/\text{s} \]
\[ k = 0.027 \text{ W/m} \cdot \text{K} \]
\[ c_p = 1000 \text{ J/kg} \cdot \text{K} \]
\[ \rho = 1.04 \text{ kg/m}^3 \]

(c) Miscellaneous:
For Part (a) of the problem statement, sensible heat effects may be neglected relative to latent heat effects.