PART A:

1. At sea floor there is no velocity normal to the ground \( \vec{v} \cdot \hat{n} = 0 \); \( \frac{\partial \phi}{\partial n} = 0 \) on \( z = -h \).

2. Streakline.

3. Pathline.

4. \( \frac{\partial \vec{v}}{\partial t} = 0 \)

5. \( \vec{v} \cdot \nabla = 0 \)

6. \[ c = -4 \]

7. \( w = 2\pi f = 2.13 \text{ rad/s} = \frac{g}{k} \) \( \Rightarrow \frac{k}{g} \) \( \Rightarrow \frac{\omega^2}{g} = k \)

- Waves are deep
- \( \omega^2 = g h \)
- \( h = 13.1 \text{ m} \)
- \( v_p = \frac{g}{2 \pi} = 4.5 \text{ m/s} \)
- \( v_g = \frac{1}{2} v_p = 2.3 \text{ m/s} \)

8. \( w = 5.6 \text{ rad/s} \)

- Still deep
- \( \omega^2 = g \frac{k}{h} \)
- \( v_p = \frac{w}{2\pi} \)
- \( v_g = \frac{v_p}{2\pi} \)

9. \( p = -\rho g z = 1000 \cdot 10 \cdot 100 = 10^6 \text{ N/m}^2 \)

10. \( p_0 = \frac{1}{2} \rho v^2 \)

11. \( p_d = -\rho \frac{\partial v}{\partial t} \)
10. Irrotational \( \nabla \times \vec{V} = 0 \) [Does] \( \checkmark \)

Continuity \( \nabla \cdot \vec{V} = 0 \) [Does Not] \( \checkmark \)

**PART B.**

"L" waves in central Atlantic.

\[
\begin{align*}
\text{deep water} & \\
\omega^2 = gk & \\
k = 0.324 \frac{\text{rad}}{\text{s}} & \\
\lambda = 9.4 \text{ m} & \\
\omega = 1.9 \text{ rad/s} & \\
a = 1 \text{ m} & \\
\end{align*}
\]

(4) \( 2a \lambda^2 = 2/19.4 = 0.103 < \frac{1}{4} \) (0.143)

\( \therefore \text{yes, linear assumption is valid but for } \theta \approx 0 \frac{\text{rad}}{\text{s}} = 0.2 \text{, not valid} \)

6. \( KH > \pi \) so if \( KH \approx \pi \) then too shallow

\( KH = 0.324 \frac{\text{rad}}{\text{s}} \approx \pi \)

\( H \approx 9.1 \text{ m} \)

\( E = \frac{1}{2} g a^2 = 5000 \text{ J/m}^2 \)

\( KE = \frac{1}{2} E = \frac{1}{4} g a^2 = 2500 \text{ J/m}^2 \)
Flow Through Nozzle

Continuity:

\[
p_1 V_1 A_1 = p_2 V_2 A_2
\]

\[
V_1 A_1 = V_2 A_2
\]

\[
V_2 = V_1 \frac{A_1}{A_2}
\]

\[
\left( \frac{A_1}{A_2} = \frac{V_2}{V_1} \right)
\]

Bernoulli's Eqn gives \( \Delta P \)

\[
p_1 + \frac{1}{2} p_1 V_1^2 + p_1 g z_1 = p_2 + \frac{1}{2} p_2 V_2^2 + p_2 g z_2
\]

\[
(P_1 - P_2) = \frac{1}{2} p_1 (V_2^2 - V_1^2) + p_1 g (z_2 - z_1)
\]

From manometer:

\[
(P_1 - P_2) = \frac{p g D}{2}
\]

\[
\frac{p g D}{2} = \frac{1}{2} c_1 \left( \frac{A_1^2}{A_2^2} - 1 \right) + p_1 g (z_2 - z_1)
\]

\[
\frac{\frac{p g D}{2} + p_1 g (z_1 - z_2)}{\frac{V_2^2}{2 c_1 V_1^2}} + 1 = \frac{A_1}{A_2}
\]
Max force is on the harbor gate:

1. Boat goes into lock @ low tide from the river into the harbor.
2. Tide rises to high tide - now the water in the chamber is 14' deep. Harbor is 14.15' deep.

Pressure on wall (take bottom @ z = 0)

Left side:

\[ P(z) = \rho g(z + H_1 - z) \]

\[ F = L \int_0^{H_1} \rho g(z + H_1 - z) \, dz \]

\[ F = \frac{1}{2} \rho g L H_1^2 \]

Acts at \( z = \frac{H_1}{3} \)

Right side:

\[ P(z) = \rho g(H_2 - z) \]

\[ F = L \int_0^{H_2} \rho g(H_2 - z) \, dz \]

\[ F = \frac{1}{2} \rho g L H_2^2 \]

@ z = \( \frac{H_2}{3} \)
Resolving Force acts to Left:

\[ F_R = (F_2 - F_1) = \frac{1}{2} p g L (H_2^2 - H_1^2) \left( \frac{kgm}{s^2} \right) \]

where
- \[ L = 22' = 6.7m \]
- \[ H_1 = 14' = 4.27m \]
- \[ H_2 = 29' = 8.84m \]

\[ F_2 = 2007 \text{ KN} \]

Note there is a moment on the gate as well!

Total Force Acts to the left. There is a force \( F_1 \) acting to the right at \( \bar{z} = \frac{14'}{3} \)

and force \( F_2 \) acting to left @ \( \bar{z} = \frac{29'}{3} \).

These forces supply a moment to the gate as shown above.

\[ F_R \cdot 2 = F_2 \cdot \bar{z}_2 - F_1 \cdot \bar{z}_1 \Rightarrow \bar{z} = 3.4 \text{ m from bottom} \]

The forces act at the center of the gate from side to side.

Design-wise any of the 3 will work.

- Design A will require the most torque to move the gate as the force on each door in B is \( \frac{1}{2} \) of one whole gate in A. B requires less space for
- Design B requires 4 motors could be expensive though hydraulic system would not be too much different than in A
- Design C is easiest to open but hardest to seal

Design through outside.