PROBLEM SET 2 SOLUTIONS

1. Recall $Y^* = \frac{(c_0 - c_1^* + I + G)}{(1-c_1)}$, $C = c_0 + c_1^*(Y-T)$, and $S = (Y-T)-C$.

   a. $Y^* = \frac{(100-0.75*40+50+40)}{(1-0.75)} = 640$
      
      $C^* = 100 + 0.75*(640-40) = 550$
      
      $S^* = -100 + (1-0.75)*(640-40) = 50$

   b. 
      
      **Change in government expenditure:**
      
      $dY^* = \frac{dG}{1-c_1} = \frac{12}{1-0.75} = 48$
      
      $dC^* = c_1^*dY^* = 0.75*48 = 36$
      
      $dS^* = dY^* - dC^* = 48 - 36 = 12$
      
      $d(G-T) = 12$

      Equilibrium income rises by 48, consumption by 36, saving by 12, and the budget deficit by 12.

      **Change in lump-sum taxes:**
      
      $dY^* = -c_1^*dT/(1-c_1) = -0.75*(-12)/(1-0.75) = 36$
      
      $dC^* = c_1^*(dY^* - dT) = 0.75*(36 - (-12)) = 36$
      
      $dS^* = dY^* - dT - dC^* = 36 - (-12) - 36 = 12$
      
      $d(G-T) = 12$

      Equilibrium income rises by 36, consumption by 36, saving by 12, and the budget deficit by 12.

      The difference in policies on equilibrium income is motivated by higher government expenditure raising income by $12 + 12*0.75 + 12*0.75^2 + \ldots$ while lower taxes raising income by $12*0.75 + 12*0.75^2 + 12*0.75^3 + \ldots$. In other words the initial effect of the tax cut is to raise disposable income, a fraction of which is spent on goods and services by consumers while higher government expenditures directly translates into a dollar for dollar increase in final output.

   c. 

      **Change in marginal propensity to consume:**
      
      $Y^* = \frac{(100-0.50*40+40+50)}{(1-0.50)} = 340$
      
      $C^* = 100 + 0.50*(340-40) = 250$
      
      $S^* = -100 + (1-0.50)*(340-40) = 50$

      The response of consumers is plausible of they expect higher taxes in the future, and thus reduce consumption now (although one would expect a smaller response due to such a small increase in the deficit).

      **Change in both marginal propensity to consume and government expenditure:**
      
      $Y^* = \frac{(100 - 0.50*40+40+50+52+50)}{(1-0.50)} = 364$
      
      $C^* = 100 + 0.50*(340-40) = 262$
      
      $S^* = -100 + (1-0.50)*(340-40) = 62$

      This example illustrates that fiscal policy which is not properly financed could potentially be ineffective if consumers respond to larger deficits by increasing saving.

2. Now $Z = C + I$ and $C = c_0 + c_1^*Y$ and $Z=Y$ defines the equilibrium

   a. 

      **The no-government equilibrium:**
      
      $Y^* = \frac{(c_0 + I)}{(1-c_1)}$
      
      $C^* = c_0 + c_1^*(c_0 + I)/(1-c_1)$
      
      $S^* = I$

   b. 

Equilibrium with output stabilization using government expenditure:
\[ Y_p = (c_0 + I + G)/(1-c_1) \]
\[ Y_p - Y^* = G/(1-c_1) \]
\[ G_p = (1-c_1)*(Y_p - Y^*) = (1-c_1)*Y_p - (c_0 + I) \]

c.
Equilibrium with output stabilization using taxes:
\[ Y_p = (c_0 - c_1*T + I)/(1-c_1) \]
\[ Y_p - Y^* = -c_1*T/(1-c_1) \]
\[ T_p = (1-c_1)*(Y^* - Y_p)/c_1 = [(c_0 + I) - (1-c_1)*Y_p]/c_1 \]

The policy rules are similar except for two parts. The rule for taxes is negative that for government expenditures and the rule for taxes is inflated by the marginal propensity to consume, implying that it will always take more taxes than government expenditures to stabilize output. This is because the multiplier for taxes is smaller than that for government expenditures for reasons mentioned in question #1.

d.
Equilibrium with no budget deficits and output stabilization:
\[ Y_p = (c_0 - c_1*T + I + G)/(1-c_1) \]
\[ \text{Impose } T = G = E \]
\[ Y_p = (c_0 + I)/(1-c_1) + E \]
\[ E_p = Y_p - Y^* = Y_p - (c_0 + I)/(1-c_1) \]

When the government is forced to balance the budget, the policy multiplier has fallen from either 1/(1-c_1) or c_1/(1-c_1) to 1 (which is simply the difference between the two multipliers, as is required by every increase in government purchases be financed by an increase in taxes). It is still possible to stabilize output, but the size of the government measured as G/Y_p, is much larger when the government operates under a balanced budget.

3. Recall \( Z_t = c_0 + c_1*(Y_t-T) + I + G \) and \( Y_{t+1} = Z_t \)
a. \[ Y_t = 500 \]
\[ Y_{t+1} = 100 + 0.75*(500-40) + 50 + 40 = 535 \]
\[ Y_{t+2} = 100 + 0.75*(535-40) + 50 + 40 = 561.25 \]
\[ Y^{ss} = (100-0.75*40+40+50)/(1-0.75) = 640 \]

There is excess demand for goods met by the depletion of inventories. Firms slowly increase production to meet last period’s excess demand, but this has a feedback effect which raises income and creates excess demand again (although less than the previous period). This process repeats until excess demand converges to zero in steady-state.

b. \[ Y_t = 640 \]
\[ Y_{t+1} = 80 + 0.75*(640-40) + 50 + 40 = 620 \]
\[ Y^{ss} = (80-0.75*40+40+50)/(1-0.75) = 560 \]

c. The policy rule is simply to increase government expenditure by the fall in autonomous consumption so that G = 60 every period. Verify this rule below:
\[ Y_{t+1} = 80 + 0.75*(640-40) + 50 + 60 = 640 = Y_t \]

d. Track the debt of the economy:
\[ D_t = 0 \]
\[ D_{t+1} = 0 + 60 - 40 = 20 \]
\[ D_{t+2} = 20 + 60 - 40 = 40 \]

The government is steadily accumulating debt in order to finance its fiscal policy. This rule is obviously not sustainable forever because as the size of the debt grows lenders will feel less confident it will be repaid, that thus less likely to lend in the first place. Obviously a temporary shock is easily reversed through fiscal policy with relatively little debt accumulation.