PROBLEM SET 1 SOLUTIONS

1. True, False, Uncertain
   a. FALSE. It is important to understand there can be differences between income and expenditure. Aggregate expenditure must be at least as large as imports whether they are final or intermediate goods. It is possible, however, that the economy actually produces nothing, and spends money either by borrowings or transfers from other countries or by reducing its savings (using income from previous periods). I am told a good example of transfers would be Tahiti. You might also think of an economy of lawyers, who obviously produce nothing at all, but are able to purchase goods by using income saved from previous periods when people mistakenly paid them money for their services. I do realize some of your parents are lawyers, so think of good lawyers who create output and bad lawyers who destroy output, so on average they produce nothing at all.

   b. UNCERTAIN. There is no change in the production of child care services. When women stayed home these services were not measured, and now they are as women hire child care workers, but the true output of child care has not changed. On the other hand, women are producing measured goods and services now, so true output could either be higher or unchanged. If the creation of a child care industry simply replaces output and employment in another industry, then there is no change in true output. On the other hand, if the child care industry does not grow at the expense of other industries (perhaps due to growth in the size of the population), then it is possible true output is higher.

   c. FALSE. Okun’s law states that the change in the unemployment rate is inversely related to excess growth rate of output, where excess growth rate of output is simply actual growth less long-run growth. Consequently, the unemployment rate is rising whenever actual growth is less than the long-run rate of growth. If the long-run growth rate is positive, then it follows there exist growth rates between zero and the long-run growth rate where the unemployment rate is rising.

   d. FALSE. The modified Phillips curve suggests that reducing the rate of inflation will cause unemployment to rise above the natural rate for some time.

2. Measuring GDP
   a. GDP_{1998} = 10*1+10*1 = 20  
      GDP_{1999} = 20*1+10*2 = 40  
   b. r_{98} GDP_{1998} = 20  
      r_{98} GDP_{1999} = 20*1+10*1 = 30  
      GDP_{def,1998} = GDP_{1998}/r_{98} GDP_{1998} = 20/20 = 1  
      GDP_{def,1999} = GDP_{1999}/r_{98} GDP_{1999} = 40/30 = 1.3333  
      %\Delta GDP = 100*(30-20)/20 = 50\%  
      %\Delta GDP_{def} = 100(1.333-1)/1 = 33\%  
   c. r_{99} GDP_{1998} = 10*1+10*2 = 30  
      r_{99} GDP_{1999} = 40  
      GDP_{def,1998} = GDP_{1998}/r_{99} GDP_{1998} = 20/30 = 0.6666  
      GDP_{def,1999} = GDP_{1999}/r_{99} GDP_{1999} = 40/40 = 1  
      %\Delta GDP = 100*(40-30)/30 = 33\%  
      %\Delta GDP_{def} = 100(1-0.6666)/0.6666 = 50\%  

3. The modified Phillips Curve
   a. Rewrite the two equations as follows:
      WS W = P(1-\alpha u+z)  
      PS W = P(1+\beta u)  
      Note in (W,u) space WS is a downward-sloping line with an intercept of P(1+z) and slope -\alpha P<0 while PS is an upward-sloping line with an intercept of P and slope \beta P>0. Equilibrium is described by the intersection of these two lines.
      Intersection of these lines occurs where WS=PS, which is mathematically characterized by the following equation, eliminating the nominal wage for the two-equation system.
      P(1-\alpha u+z)=P(1+\beta u)  
      Solving for the equilibrium unemployment rate  
      u = [P(1+z)-P]/[\beta P+\alpha P]
And finally inserting this unemployment rate into wage-setting yields equilibrium nominal wages
\[ W = P(1+\beta u) = P(1+\beta)[P_e(1+z)-P]/[\beta P+\alpha P^e] \]

There is unemployment in this labor market due to the presence of unemployment benefits \((z>0)\) which drives up the wage-demands of workers by increasing their opportunity cost of working.

b. The natural rate of unemployment is simply constructed by imposing \(P^e=P\) in the equation for equilibrium unemployment above.
\[ u_n = z/(\beta+\alpha) \]
As unemployment benefits increase, the natural rate of unemployment increases. This happens as the wages demanded by workers increase for every rate of unemployment.

c. In \((W,u)\) space the dynamics are simple to describe. The unexpected increase in the level of prices shifts the PS curve up (check the intercept) and increases its slope. There is no change in the WS curve this period as price expectations depend on last period’s prices, which have not changed. Note that the nominal wage is higher and the unemployment rate is lower than the natural rate. Next period, the price level remains at its new level while price expectations are updated, shifting the WS up (check the intercept again) and increases its negative slope. The rate of unemployment returns to its natural level, while nominal wages are higher.

Mathematically, describe the process for prices as such:
\[ P_{t+1} = P_{t+1}, P_{t+1} = P_{t+1} = 1+\varepsilon \text{ for } \varepsilon>0 \]
This implies a process for price expectations give \(P^e_t=P_{t+1}\) as follows:
\[ P_{t+1}^e = P_{t+1}^e = P_{t+1}^e = 1+\varepsilon \text{ for } \varepsilon>0 \]

Break up the dynamics into three periods.
For time\(t\leq t\) we have \(P=P^e=1\), implying \(u=u_n\) and \(W=(1+\beta u_n)\)
For time\(t=t+2\) we have \(P=P^e=1+\varepsilon\), implying \(u=u_n\) and \(W=(1+\varepsilon)(1+\beta u_n)\)
Finally for time\(t=t+1\), we have \(P=1+\varepsilon\) but \(P^e=1\), implying
\[ u = (z-\varepsilon)/[\beta(1+\varepsilon)+\alpha] \text{ and } W = (1+\varepsilon)(1+\beta(z-\varepsilon)/[\beta(1+\varepsilon)+\alpha]) \]
At time \(t=1\), prices are higher than expected, so the real wage firms pay to workers has fallen, prompting the increase in employment and fall in unemployment. The next period, however, worker’s expectations catch up with prices, and all returns to normal. The new nominal wage should adjust so that the real wage is the same as before the increase in prices.

d. \(P_P=P^e(1+z\cdot u_n)/(1+\beta u_n)\)
\(1+\Pi_l=(1+\Pi)^e(1+z\cdot u_n)/(1+\beta u_n)\)
\(\ln(1+\Pi_l)=\ln(1+\Pi)^e+\ln(1+z\cdot u_n)-\ln(1+\beta u_n)\)
\(\Pi_l=\Pi_l^e+z\cdot(\alpha+\beta)u_l\)
\(\Pi_l=\Pi_l^e+z\cdot(\alpha+\beta)u_l\)
e. \(d\Pi=0\) implies \(u_n=z/(\alpha+\beta)\)
The natural rate of unemployment was defined as equilibrium unemployment rate when prices were what workers expected them to be. The modified Phillips curve is really nothing more than price-setting and wage-setting, so it should not be surprising that the rate of unemployment when inflation is equal to expected inflation is the natural rate of unemployment.
Simplifying the equation using \(z=(\alpha+\beta)u_n\) yields the following
\[ d\Pi_l=-(\alpha+\beta)(u_l-u_n) \]
e. Solve the modified Phillips curve for the unemployment rate
\[ u_l = u_n-d\Pi_l/(\alpha+\beta) \]
Thus if \(d\Pi_l=-10\%\) then it follows that the unemployment rate will rise above the natural rate of unemployment by \(10/(\alpha+\beta)\) percentage points.
f. The parameter \(\alpha\) has the interpretation as the sensitivity of nominal wage demands to the unemployment rate. An increase in this sensitivity reduces the natural rate of unemployment, and makes the modified Phillips curve steeper in \((d\Pi,u)\) space. An increase in \(\alpha\) also reduces the amount of unemployment over and above the natural rate for the given reduction in inflation. What is driving this result is that the wage demands of workers are becoming more sensitive to the unemployment rate, fixing price expectations, so the ability of firms to take advantage of surprises in prices is somewhat offset by the increased bargaining power workers have when the unemployment rate
falls. On the other hand, workers have much less bargaining power when the unemployment rate rises (while their wage demands are higher when the unemployment rate falls, their wage demands are now lower when the unemployment rate rises). This implies that it is much easier for firms to reduce wages when the unemployment rate rises, and consequently it is much easier to reduce their prices.