Problems 1, 2, 3 (only part a) will be graded. Problem 4 is worth doing. It’s a good exercise; even though we didn’t explicitly do career concern models, Diamond’s paper is in that general category.

Problem 1.

In Tirole’s paper on “Collective Reputations” (ReStud ’96), show that the suggested mixed strategy equilibrium in footnote 12 indeed is an equilibrium under the conditions specified.

Problem 2.

Consider an infinite horizon version of Diamond’s model of reputation acquisition (JPE ’91).

a. Analyze the equilibrium for the case where strategic types are of zero measure ($f_{BG} = 0$), while bad and good types have strictly positive measure ($f_B > 0$, $f_G > 0$).

b. Suppose strategic types have strictly positive measure. Will there always (or ever) exist an equilibrium in pure strategies? (I don’t know the answer, but believe it depends on parameter values; in particular, $f_{BG}$).

Problem 3.

Prove Proposition 5 in Milgrom-Roberts “Relying on Information from Interested Parties,” (Rand ‘86).
Problem 4.

Consider the following variant of a career concern model:

There are two identical periods. After the second period, the manager retires. The output of the manager in period \( t \) is:

\[
y_t = \mu + e_t \varepsilon_t + (1-e_t)\theta_t
\]

where \( \mu \) is the manager's (unknown) ability, \( e_t \in [0,1] \) is the manager's (unobserved) action, \( \varepsilon_t \) is an unobserved stochastic return term and \( \theta_t \) is an (ex post) observed stochastic return. You may think of the manager's action as a decision to allocate a dollar between a firm-specific project, which returns \( \varepsilon_t \) and a market project, which returns \( \theta_t \). The allocation is only known to the manager.

The market pays the manager his expected value in each period, \( w_t = E(y_t|I_t) \), which depends on the market's information \( I_t \) and its expectation of the manager's action. Assume that the market's (and the manager's) initial beliefs (in period 1) about ability are such that \( \mu \) is normally distributed with mean \( m_1 \) and precision (the inverse of variance) \( h_1 \). Beliefs in the second period are updated based on inferences about \( e_1 \), the observed outcome \( y_1 \) and the observed market return \( \theta_1 \).

Assume that the returns \( \varepsilon_t \) and \( \theta_t \) are independent across time as well as of each other. Each is normally distributed with zero mean and with precisions \( h_\varepsilon \) and \( h_\theta \), respectively.

The manager is strictly risk averse. His preferences can be described by:

\[
E_t (E(w_t) - r \text{Var}(w_t)),
\]

where \( w_t \) is his income in period \( t \), \( E \) is the expectation operator and \( \text{Var} \) is the variance operator. The coefficient of risk aversion is \( r > 0 \). Note that there is no cost associated with choosing \( e_t \); instead, \( e_t \) is constrained to lie in the interval \([0,1]\).

a. Write down the equations that characterize a rational expectations equilibrium for this model.

b. Show that in the rational expectations equilibrium, the manager will necessarily choose the first-period allocation \( e_1 = 0 \); i.e., he will invest all the money in the market project.

c. Would the conclusion in (ii) be altered if we instead assumed that the firm-specific project had an expected return \( E(\varepsilon_t) = 1 \)? Why or why not?

d. Would the answer in part b be different if the manager knew his/her own type?