1 Tirole

There is a typo in the footnote. When faced with an agent with a spotless record, principals offer task 1 with probability $\theta$ and task 2 with probability $1 - \theta$.

We need to check (a) we can indeed construct the strategy in the way specified there, and (b) the strategy profile is actually an equilibrium.

(a) As for construction, first we need to check $\theta$ defined there actually lies between 0 and 1. This is obvious from Assumption 3. Next, we need to show that there exist a strategy of agents such that the overall probability of honest behavior $v$ indeed satisfies

$$v(H - h) + (1 - v)(D - d) = 0.$$ 

Let $v^*$ be the value of $v$ which satisfies this equality. Suppose an agent behaves with probability $\eta$ if he hasn’t cheated before. (We are going to show that the condition above is satisfied for some $\eta \in [0, 1]$.) Among opportunistic agents, the proportion of agents who haven’t cheated before is

$$\frac{1 + \lambda \eta + \lambda^2 \eta^2 + \cdots}{1 + \lambda + \lambda^2 \cdots} = \frac{1 - \lambda}{1 - \lambda \eta},$$

and thus the proportion of those who have cheated at least once is

$$1 - \frac{1 - \lambda}{1 - \lambda \eta} = \frac{\lambda(1 - \eta)}{1 - \lambda \eta}.$$ 

Among opportunistic agents who have cheated at least once, the proportion of agents who has cheated exactly $k$ times is

$$(1 - \lambda)\lambda^k.$$ 

Therefore, the proportion of spotty agents among opportunistic agents who have cheated at least once is

$$(1 - \lambda)(x_1 + \lambda x_2 + \lambda^2 x_3 + \cdots),$$
and the proportion of spotless agents is
\[
(1 - \lambda)(1 - x_1 + \lambda(1 - x_2) + \lambda^2(1 - x_3) + \cdots) = \frac{Y - (1 - \lambda)}{\lambda}.
\]
Therefore, the overall probability of honest behavior \( v \) is
\[
v(\eta) = \frac{\alpha + \gamma(\frac{1 - \lambda}{1 - \lambda\eta})\eta}{\alpha + \gamma(\frac{1 - \lambda}{1 - \lambda\eta}) + \gamma\frac{\lambda(1 - \eta)}{1 - \lambda\eta} - \frac{1 - \lambda}{\lambda} + \beta Y}.
\]
We want to check there exists \( \eta \in [0, 1] \) such that \( v(\eta) = v^* \). To see this, first observe
\[
v(1) = \frac{\alpha + \gamma}{\alpha + \gamma + \beta Y},
\]
and from Assumption 2, \( v(1) > v^* \). Second,
\[
v(0) = \frac{\alpha}{\alpha + \gamma(1 - \lambda) + \gamma\lambda\frac{Y - (1 - \lambda)}{\lambda} + \beta Y} = \frac{\alpha}{\alpha + (\gamma + \beta)Y},
\]
and from Assumption 4, \( v(1) < v^* \). Finally, since \( v(\eta) \) is a continuous function, there exists \( \eta \in [0, 1] \) such that \( v(\eta) = v^* \) from the intermediate value theorem.

(b) To show that the suggested strategy profile is actually an equilibrium, we need to check four conditions:

1. A principal is indifferent between offering task 1 and 2 when meeting a spotless agent.
2. A principal is better off by offering task 2 when meeting a spotty agent.
3. An agent is indifferent between behaving and cheating if he hasn’t cheated before.
4. An agent is better off by cheating if he has cheated before.

Now let’s show each condition is satisfied.

1. This is obvious from the definition of \( v \).
2. A spotty agent is either an opportunistic agent who has cheated before or a dishonest agent. Both will cheat for sure, so it is better to offer task 2.
3. It is sufficient to prove that an agent is indifferent between behaving every period from now and cheating every period from now. (Think yourself why.)
The payoff from behaving every period is

$$0 + (\delta + \delta^2 + \cdots)(\theta B + (1 - \theta)b) = \frac{\delta(\theta B + (1 - \theta)b)}{1 - \delta}.$$ 

The payoff from cheating every period is

$$G + \delta((1 - x_1)(\theta B + (1 - \theta)b) + x_1 b + G)$$

$$+ \delta^2((1 - x_2)(\theta B + (1 - \theta)b) + x_2 b + G) + \cdots$$

$$= G + \delta((\frac{1}{1 - \delta} - Z)(\theta B + (1 - \theta)b) + Zb + \frac{G}{1 - \delta})$$

$$= \frac{G}{1 - \delta} + \delta((\frac{1}{1 - \delta} - \theta B + (1 - \theta)b - Z\theta(B - b)).$$

From the definition of $\theta$, these are the same.

4. Obvious from part 3 above and Appendix 1 of the paper.

2 Diamond

(a)

Since there is no strategic type, monitoring never happens. No debt contract is offered to a borrower without clean history, because a borrower without clean history is of bad type for sure and thus there is no gains from trade.

Consider an equilibrium where a debt contract is offered to borrowers with clean history. In equilibrium, all borrowers with clean history takes this because the good type must be taking this and the bad type with clean history also has an incentive to take this if the good type wants to take the debt offer. In this kind of equilibrium, the zero profit condition of lenders implies

$$r_t(f_t^G + (1 - f_t^G)\pi) = R,$$

and thus

$$r_t = \frac{R}{f_t^G + (1 - f_t^G)\pi}.$$ 

The population of borrowers with clean history at time $t$ is $f_t^G + (1 - f_t^G)\pi^{t-1}$, i.e., the sum of the population of good type and the population of bad type who has never failed before. Therefore

$$f_t^G = \frac{f_t^G}{f_t^G + (1 - f_t^G)\pi^{t-1}}.$$ 

This implies $f_t^G = f_1^G$. $f_t^G$ is increasing and converges to 1. Therefore,

$$r_1 = \frac{R}{f_1^G + (1 - f_1^G)\pi},$$
is decreasing and converges to $R$.

The good type is willing to take the debt offer every period if and only if

$$\sum_{t=1}^{\infty} d^t(G - r_t) \geq 0.$$ 

Otherwise, the good type is better off not taking the debt contract at all, in which case the equilibrium is such that no debt contract is offered at any time.

(b)

To be completed

3 Milgrom and Roberts

First let’s show the existence. Let $d^*(\omega)$ the full-information decision in state $\omega$, i.e.,

$$d^*(\omega) = \arg\max_{d \in \Delta} u(Z, x_d, d).$$

The following strategy-belief pair makes a sequential equilibria.

- Every interested party reports $A_i = Z$ and $D_i$ such that $d^*(\omega) \in D_i$.

- The decision maker adopts the skeptical strategy.

- Off equilibrium path beliefs which are not specified by skeptical strategy can be anything.

An interested party doesn’t have an incentive to unilaterally deviate because in equilibrium the decision maker is choosing $d^*(\omega)$ and the interested party can never induce the decision maker to choose something else. This is because the decision maker is already having full information about $d^*(\omega)$ from other agents and thus can never go down in ranking by getting any additional information. The skeptical strategy is obviously sequentially rational and satisfies the Bayes’ rule.

Next let’s show that all equilibrium with the skeptical strategy induces $d^*(\omega)$.

The assumption stated in the proposition is equivalent to saying “for all $\omega$ and for all $d \neq d^*(\omega)$, there exists an interested party $i$ such that $d^*(\omega) \succ_i d$.”

Suppose there exists an equilibrium with the skeptical strategy which induces $d \neq d^*(\omega)$. Then the interested party $i$ such that $d^*(\omega) \succ_i d$ has an incentive to report $A_i = Z$ and $D_i$ such that $d^*(\omega) \in D_i$, because then the decision maker will choose $d^*(\omega)$ if she follows the skeptical strategy. Contradiction.
4 Career Concern

- $t = 1, 2$
- $y_t = \mu + e_t \epsilon + (1 - e_t) \theta_t$
- $\mu$: ability
- $e_t$: unobserved stochastic return on firm specific project
- $\theta_t$: observed stochastic return on market project
- $w_t = E(y_t|I_T)$
- $\mu \sim N(m, \frac{1}{h_1})$
- $\epsilon_t \sim N(0, \frac{1}{h_{\epsilon}})$
- $\theta_t \sim N(0, \frac{1}{h_{\theta}})$
- The manager’s preference: $\sum_t E[w_t] - r \text{Var}[w_t]$

(a)

$$w_1 = E(y_1) = m_1$$

Let $e_1^*$ be the equilibrium effort level in period 1. In first period,

$$y_1 = \mu + e_1^* \epsilon_1 + (1 - e_1^*) \theta_1.$$ 

The market observes

$$z_1 = y_1 - (1 - e_1^*) \theta_1 = \mu + e_1^* \epsilon_1$$

and therefore

$$w_2 = E(y_2|I_2) = E(\mu|I_2)$$

$$= \frac{h_1 m_1 + h_{\epsilon} e_2 z_1}{h_1 + h_{\epsilon}}$$

$$= \alpha m_1 + (1 - \alpha) z_1$$

where

$$z_1 = y_1 - (1 - e_1^*) \theta_1,$$

$$\alpha = \frac{h_1}{h_1 + h_{\epsilon}}.$$ 

And,

$$e_1^* = \text{argmax } E(w_2) - r \text{Var}(w_2).$$

(1), (2) and (3) characterize a rational expectations equilibrium.
Now we explicitly solve for (3). Notice that by choosing \( e_1 \), the agent can affect the outcome but cannot change the market’s expectation, namely \( \hat{e}_1 \).

The incentive constraint (3) says that if the market’s expectation is \( \hat{e}_1 = e^*_1 \), then the agent has an incentive to choose \( e_1 = e^*_1 \).

\[
E(w_2) - r \text{Var}(w_2) = E(\alpha m_1 + (1 - \alpha)z_1) - r \text{Var}(\alpha m_1 + (1 - \alpha)z_1) \\
= m_1 - r(1 - \alpha)^2 \text{Var}(z_1) \\
= m_1 - r(1 - \alpha)^2 \text{Var}(y_1 - (1 - e^*_1)\theta_1) \\
= m_1 - r(1 - \alpha)^2 \text{Var}(\mu + e_1\epsilon_1 + (1 - e_1)\theta_1 - (1 - e^*_1)\theta_1) \\
= m_1 - r(1 - \alpha)^2 \left( \frac{1}{h_1} + \frac{e^2_1}{h_\epsilon} + \frac{(e^*_1 - e_1)^2}{h_\theta} \right)
\]

- If \( e^*_1 = 0 \),
  \[\arg\max_{e_1} E(w_2) - r \text{Var}(w_2) = 0\]
  and condition (2) is satisfied.

- If \( e^*_1 > 0 \),
  \[0 < \arg\max_{e_1} E(w_2) - r \text{Var}(w_2) < e^*_1\]
  and condition (2) is not satisfied.

Therefore, we must have \( e^*_1 = 0 \).

Here, the agent’s first period choice doesn’t affect the expected value of the second period wage, thus his only concern is to minimize its variance.

(c)

Now assume \( E(\epsilon_1) = 1 \).

\[
w_1 = E(y_1) = m_1 + e^*_1
\]  \hspace{1cm} (4)

In first period,
\[
y_1 = \mu + e^*_1\epsilon_1 + (1 - e^*_1)\theta_1
\]
and the market observes
\[
z_1 = y_1 - e^*_1 - (1 - e^*_1)\theta_1 = \mu + e^*_1(\epsilon - 1).
\]
Therefore,

\[ w_2 = E(y_2|I_2) = E(\mu|I_2) + e_2^* \]

\[ = \frac{h_1 m_1 + \frac{h_2}{e_2^*} z_1}{h_1 + \frac{h_2}{e_1^*}} + e_2^* \]

\[ = \alpha m_1 + (1 - \alpha) z_1 + e_1^*, \quad (5) \]

where

\[ z_1 = y_1 - e_1^* - (1 - e_1^*) \theta_1 \]
\[ \alpha = \frac{h_1}{h_1 + \frac{h_2}{e_1^*}}. \]

And,

\[ e_1^* = \arg \max E(w_2) - r \text{Var}(w_2). \quad (6) \]

Now (4), (5) and (6) characterize a rational expectation equilibrium.

For simplicity, consider an equilibrium such that \( e_2^* = 1 \) after any history, because it is efficient. (There are other equilibria where different levels of second period effort are chosen.) Now,

\[ E(w_2) - r \text{Var}(w_2) \]
\[ = E(\alpha m_1 + (1 - \alpha) z_1 + e_2^*) - r \text{Var}(\alpha m_1 + (1 - \alpha) z_1) \]
\[ = \alpha m_1 + (1 - \alpha)(m_1 + e_1 - e_1^*) + 1 - r(1 - \alpha)^2 \text{Var}(z_1) \]
\[ = m_1 + (1 - \alpha)(e_1 - e_1^*) + 1 - r(1 - \alpha)^2 \left( \frac{e_1^2}{h_\epsilon} + \frac{(e_1^* - e_1)^2}{h_\theta} + \frac{1}{h_1} \right). \]

If \( e_1^* = 0 \),

\[ \arg \max_{e_2^*} E(w_2) - r \text{Var}(w_2) > 0 \]

because

\[ \frac{\partial}{\partial e_1} E(w_2) - r \text{Var}(w_2)|_{e_1=0} = 1 - \alpha > 0 \]

and so condition (6) is not satisfied. Therefore, we must have \( e_1^* > 0 \).

This is because now \( e_1 \) affects not only \( \text{Var}(w_2) \) but also \( E(w_2) \). Now, the manager has an incentive to pretend to be of high ability \( \mu \) by choosing high \( e_1 \), although in equilibrium the manager can not cheat.

(d)

Things become very different. Hard to solve the model.