Problem Set 3: Contract Law

Assigned: Oct. 5
Answers: Oct. 20
Midterm: Oct. 24

Part A: Game Theory Review

Question A1. Players A and B are engaged in a coin-matching game. Each shows a coin as either heads or tails. If the coins match, B pays A $1. If they differ, A pays B $1.

(a) Write down the payoff matrix for this game and show that it does not contain a Nash equilibrium in pure strategies.

(b) Find the mixed-strategy Nash equilibrium and compute the players’ equilibrium expected surpluses

Question A2. The game of “chicken” is played by two rebel teens, Fonzi and Pinkie, who speed toward each other on a single lane road. The first to veer off is branded the chicken whereas the one who doesn’t turn gains peer group esteem. Of course, if neither veers, both die in the resulting crash. Payoffs to the chicken game are provided in the following table.

\[
\begin{array}{c|cc}
& \text{Chicken} & \text{Not Chicken} \\
\hline
\text{Chicken} & 2,2 & 1,3 \\
\hline
\text{Not Chicken} & 3,1 & 0,0 \\
\end{array}
\]

(a) Compute the pure-strategy Nash equilibria in this game.

(b) Compute the mixed-strategy equilibrium and the players’ expected equilibrium payoffs.

(c) Consider a new game with a more complicated timing structure. First, Fonzi gets to choose his course, either chicken or not chicken. Then his steering wheel detaches and he is committed to his course. Pinkie observes if Fonzi is coming toward her or not and then makes her decision whether to chicken out or not.

i. Write down the strategies for this more complicated game. (Hint: Pinkie’s strategies are more complicated, being contingent on the actions of Fonzi).

ii. Write down the payoff matrix for this more complicated game.

iii. What are the Nash equilibria?

iv. Which Nash equilibria are subgame perfect?
Part B: Applications to Contract Law

Question B1. If McGee has a house built on his lot, he obtains a surplus of \( v \). A contractor’s cost of building the house is \( c \). The contractor promises to build the house for a price of \( p \), where \( v > p > c > 0 \). Supposing the promise is not an enforceable contract, and that the players move simultaneously, we have the following normal form:

<table>
<thead>
<tr>
<th>Contractor</th>
<th>Build</th>
<th>Don’t Build</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pay McGee</td>
<td>( v - p, p - c )</td>
<td>( -p, p )</td>
</tr>
<tr>
<td>Don’t Pay McGee</td>
<td>( v, -c )</td>
<td>( 0,0 )</td>
</tr>
</tbody>
</table>

(a) Compute the pure-strategy Nash equilibria in this game. What’s special about the equilibrium strategies?

(b) Consider a sequential version of the game in which the contractor’s decision of whether or not to build comes first, and second McGee makes the decision of whether or not to pay. Compute the Nash equilibria and subgame perfect Nash equilibria.

(c) Find the equilibria in an alternative sequential version of the game in which McGee moves first, and then the contractor moves.

(d) Return to the simultaneous game, but suppose the promise is part of an enforceable contract. Specifically, suppose the contract specifies penalties for breach for McGee \( (B_m) \) if he doesn’t pay and for the contractor \( (B_c) \) if it doesn’t build:

<table>
<thead>
<tr>
<th>Contractor</th>
<th>Build</th>
<th>Don’t Build</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pay McGee</td>
<td>( v - p, p - c )</td>
<td>( B_c - p, p - B_c )</td>
</tr>
<tr>
<td>Don’t Pay McGee</td>
<td>( v - B_m, B_m - c )</td>
<td>( B_c - B_m, B_m - B_c )</td>
</tr>
</tbody>
</table>

At what levels do \( B_m \) and \( B_c \) need to be set to ensure that McGee pays and the contractor builds in equilibrium?

Question B2. Players A and B exchange a “promise for a promise.” Let \( \alpha \) be the probability that neither player reneges on her promise and \( 1 - \alpha \) be the probability that both players renege. (Note that this implies the probability one reneges while the other doesn’t is zero.) Both players invest in reliance: player A’s expenditure is \( r_a \) and B’s is \( r_b \). If the promises are kept, A earns gross surplus \( 4r_a^{2/4}r_b^{1/4} \), and B earns gross surplus \( 4r_a^{1/4}r_b^{2/4} \). Thus, players benefit from both their own reliance and the other player’s.
(a) Suppose that the promises are not enforceable in contracts, and this implies $\alpha = 0$. Compute the equilibrium levels of reliance.

(b) Suppose that promises are enforceable in contracts, and this implies $\alpha = 1$. Compute the equilibrium levels of reliance.