Problem 1: Efficiency wages vs. subjective bonuses

Consider the following stage game of an efficiency-wage model. The agent can choose either high effort, \( a_H \), or low effort, \( a_L \). High effort yields high output, \( y \), with probability one, whereas low effort yields high output with probability \( p \) but zero output with probability \( 1-p \). The agent is risk-neutral, with payoff \( w - c(a) \), where \( w \) is the wage earned and \( c(a_H) = c > c(a_L) = 0 \). The principal is risk-neutral, with payoff equal to profit—namely, output minus wages.

The timing of this stage game is: (1) the principal offers the wage \( w \); (2) the agent accepts or rejects (in favor of alternative employment with payoff \( U_0 \)); if the agent accepts then (3) the principal pays \( w \); (4) the agent chooses \( a_H \) or \( a_L \) (but the principal does not observe this choice); (5) output is observed by the principal and the agent (but not by a court); and (6) if output is low (0) then the agent is fired, earning \( U_0 \) every period thereafter. Assume that \( y - c > U_0 > p y \), so that high effort is efficient. Finally, let the interest rate be \( r \).

State trigger strategies that would achieve high effort and high output in every period, and that are a subgame-perfect Nash equilibrium of the repeated game if the following condition holds:

\[
(*) \quad y - c \geq U_0 + \frac{r}{1-p} c.
\]

Now consider the following timing of a new stage game, to be played in the same economic environment as above (i.e., under the same assumptions about feasible actions, the relationship between actions and outputs, and so on): (1) the principal offers the contract \( (s, b) \); (2) the agent accepts or rejects (in favor of alternative employment with payoff \( U_0 \)); if the agent accepts then (3) the principal pays the salary \( s \); (4) the agent chooses \( a_H \) or \( a_L \) (but the principal does not observe this choice); (5) output is observed by the principal and the agent (but not by a court); and (6) if output is high then the principal chooses whether or not to pay the bonus \( b \).

State trigger strategies that would achieve high effort, high output, and a bonus paid in every period. Show that these trigger strategies are a subgame-perfect Nash equilibrium of the repeated game if \((*)\) holds.
Problem 2: Stationary relational contracts

Prove Theorem 2 from Levin (2003), for the moral-hazard version of Levin’s model that we discussed in class: If there is an optimal relational contract then a stationary relational contract is optimal.

Problem 3: Relational Contract Meets Multitask

This problem (eventually) concerns objective and subjective performance measurements in a multi-task relational incentive problem.

In each period, the environment is as follows (where time subscripts are omitted for simplicity). The value to the Principal from the Agent’s actions \((a_1, a_2)\) is \(y = y_H \) or \(y_L \) \((< y_H)\), where \(y\) is observable but not contractible. The probability that \(y = y_H \) is \(f_1a_1 + f_2a_2\), where \(f_1\) and \(f_2\) are non-negative and small enough that \(f_1a_1 + f_2a_2 < 1\). The Agent’s cost function is \(c(a_1, a_2) = [a_1^2 + a_2^2] / 2\). If \(w\) is the total compensation paid to the Agent in a given period then the Principal’s payoff in that period is \(y – w\) and the Agent’s is \(w - c(a_1, a_2)\). Both parties are risk-neutral, have deep pockets, and share the discount rate \(r\). The Principal’s reservation payoff is \(\pi_0\) in each period and the Agent’s is \(U_0\), where \(\pi_0 + U_0 > y_L\).

(a) What is the first-best action vector?

(b) Consider the infinitely repeated game in which the stage game has the following timing: (i) Principal and Agent can exchange money; (ii) Agent chooses actions (but Principal cannot observe them); (iii) \(y\) is publicly observed; (iv) Principal and Agent can exchange money. Specify trigger strategies that, if played, will yield the first-best. (For notational consistency in what follows, use a subjective bonus scheme that pays \(w = s\) if \(y = y_L\) but \(w = s + B\) if \(y = y_H\).) For what values of \(r\) (given the other parameters) are your strategies a subgame-perfect Nash equilibrium of the repeated game?

Now enrich the stage game to include the performance measure \(p = p_H \) or \(p_L \) \((< p_H)\), where \(p\) is contractible. The probability that \(p = p_H \) is \(g_1a_1 + g_2a_2\), where \(g_1\) and \(g_2\) are non-negative and small enough that \(g_1a_1 + g_2a_2 < 1\).

(c) Consider the one-shot agency problem (i.e., not yet a repeated game) in which the Principal’s payoff is \(y – w\) and the Agent’s is \(w - c(a_1, a_2)\), where \(y\) is not contractible but \(p\) is. Consider the incentive contract \(w = s\) if \(p = p_L\) but \(w = s + b\) if \(p = p_H\). What is the efficient value of \(b\)? Let \(b^*\) denote the efficient value of \(b\) and let \(E\pi(s, b^*)\) denote the resulting expected payoff to the Principal when the salary is \(s\). Suppose that the parties determine \(s\) via Nash bargaining, where the Principal’s bargaining power is \(\alpha \in (0, 1)\).
Denote the resulting salary by \( s_\alpha \), the Principal’s expected payoff by \( E\pi(s_\alpha, b^*) \), and the Agent’s by \( EU(s_\alpha, b^*) \).

(d) Now consider the infinitely repeated game in which the stage game has the following timing: (i) Principal and Agent can contract on \( p \); (ii) Principal and Agent can exchange money; (iii) Agent chooses actions (but Principal cannot observe them); (iv) \( y \) and \( p \) are publicly observed; (v) contracts based on \( p \) are enforced; (vi) Principal and Agent can exchange money. Assume that if reneging occurs in the repeated game then the parties will play the efficient one-shot contract from (c) forever after, where the parties’ expected payoffs are \( E\pi(s_\alpha, b^*) \) and \( EU(s_\alpha, b^*) \). Specify trigger strategies that, if played, will yield the first-best. For what values of \( r \) (given the other parameters) are your strategies a subgame-perfect Nash equilibrium of the repeated game? Are these values of \( r \) higher or lower than in (b), and why?

(e) Continue to consider the repeated game from (d), including the assumption of efficient one-shot contracting after reneging. If \( r \) is sufficiently high that the first-best cannot be achieved, it is natural to consider other relational contracts that attempt to outperform the efficient agency contract in (c). Consider the following incentive scheme: \( w = s \) if \( p = p_L \) and \( y = y_L \), \( w = s + b \) if \( p = p_H \) and \( y = y_L \), \( w = s + B \) if \( p = p_L \) and \( y = y_H \), and \( w = s + \beta \) if \( p = p_H \) and \( y = y_H \). Suppose that \( f_1 = g_1 > 0 \), \( f_2 = 0 \), and \( g_2 > 0 \). Are there finite values of \( r \) such that the only relational-contract outcome is the trivial one, which replicates the efficient agency contract from (c)? Why or why not?