Problem Set 5
Due Monday, April 12

1. Price Risk
Suppose that in 3 months the cost of a pound of Colombian coffee will be either $1.25 or $2.25. The current price is $1.75 per pound.

(a) What are the risks faced by a hotel chain who is a large purchaser of coffee?
(b) What are the risks faced by a Colombian coffee farmer?
(c) Will each of the parties be made better off by entering into a futures contract that fixes the price of coffee at $1.75? Why or why not?
(d) If the delivery price of coffee turns out to be $2.25, should the farmer have forgone entering into a futures contract? Why or why not?

2. Stock Insurance
It is now five years after you graduated. Because of the knowledge you acquired in 15.415, you have become fabulously wealthy and receive many sales pitches offering financial products. One of your friends (who did not take 15.415) approaches you with the following sales pitch:

Our clients are investors who sign guarantees that they will hold stock in XYZ Corporation for a fixed period of time, say 3 months. At that time, we will give you $1 for each $1 that the stock price falls below $100 or nothing if the stock price is above $100. We charge $2 per share of stock for this service.

You know that the current stock price of XYZ Corporation is $120, and the current price of a call option with exercise price $100 is $25. XYZ stock pays no dividends, and the amount of interest earned on a $99 investment for 3 months is $1.

Decide whether you should take your friend’s offer and carefully explain why. Can you make your decision independently of your preferences for risk? Why or why not?

3. Butterfly Spreads
A butterfly spread is a combination of a option positions that involves three strike prices. To create a butterfly spread, a trader purchases an option with a low strike
price and an option with a high strike price and sells two options with an intermediate strike price. For this problem, assume that the intermediate strike price is halfway between the low and the high strike prices and that the options are European. Denote the intermediate strike price by $X$, the low strike price by $X - a$, and the high strike price by $X + a$.

(a) Graph the payouts at maturity of the butterfly spread in which the underlying options are call options. Holding the intermediate strike price fixed, what happens to the payouts as the low and high strike price converge to the intermediate price?

(b) Suppose that a trader purchases a butterfly spread (using call options) for which the intermediate strike price equals to today’s stock price. Based only on this trade, what is the trader’s view of the future direction of the market?

(c) Use put-call parity to show that the cost of a butterfly spread created from European calls is identical to the cost of a butterfly spread created from European puts.

4. Profit Diagrams

Draw a diagram showing an investor’s profit and loss with the terminal stock price for a portfolio consisting of:

(a) One share of stock and a short position in one call option
(b) Two shares of stock and a short position in one call options
(c) One share of stock and a short position in two call options
(d) One share of stock and a short position in four call options

You should take into account the cost from purchasing the stock and revenue from selling the calls. For simplicity ignore discounting when combining these costs and revenues with the terminal payoff of the portfolio. For simplicity also assume that the current stock price is equal to the strike price, $X$, of the call. Denote the current call price by $c^E$, and the terminal stock price by $S_T$. 