Routing in Data Networks

Eytan Modiano
Packet Switched Networks

Messages broken into Packets that are routed To their destination
Routing

• Must choose routes for various origin destination pairs (O/D pairs) or for various sessions
  
  – Datagram routing: route chosen on a packet by packet basis
    
    Using datagram routing is an easy way to split paths
  
  – Virtual circuit routing: route chosen a session by session basis
  
  – Static routing: route chosen in a prearranged way based on O/D pairs
Broadcast Routing

- Route a packet from a source to all nodes in the network

- Possible solutions:
  - Flooding: Each node sends packet on all outgoing links
    Discard packets received a second time
  - Spanning Tree Routing: Send packet along a tree that includes all of
    the nodes in the network
A graph $G = (N,A)$ is a finite nonempty set of nodes and a set of node pairs $A$ called arcs (or links or edges).
Walks and paths

- A walk is a sequence of nodes \((n_1, n_2, \ldots, n_k)\) in which each adjacent node pair is an arc.
- A path is a walk with no repeated nodes.

Walk \((1,2,3,4,2)\)  Path \((1,2,3,4)\)
Cycles

• A cycle is a walk \((n_1, n_2,...,n_k)\) with \(n_1 = n_k\), \(k>3\), and with no repeated nodes except \(n_1 = n_k\).

Cycle \((1,2,4,3,1)\)
A graph is connected if a path exists between each pair of nodes.

An unconnected graph can be separated into two or more connected components.
Acyclic graphs and trees

• An acyclic graph is a graph with no cycles.

• A tree is an acyclic connected graph.

\[ \begin{array}{c}
   \text{1} & \text{2} & \text{3} & \text{4} \\
   \text{Acyclic, connected} & \text{unconnected} & \text{not tree} & \\
   \text{1} & \text{2} & \text{3} & \\
   \text{Cyclic, not tree} \\
\end{array} \]

• The number of arcs in a tree is always one less than the number of nodes
  
  Proof: start with arbitrary node and each time you add an arc you add a node ⇒ N nodes and N-1 links. If you add an arc without adding a node, the arc must go to a node already in the tree and hence form a cycle.
Subgraphs

- $G' = (N', A')$ is a subgraph of $G = (N, A)$ if
  - 1) $G'$ is a graph
  - 2) $N'$ is a subset of $N$
  - 3) $A'$ is a subset of $A$

- One obtains a subgraph by deleting nodes and arcs from a graph
  - Note: arcs adjacent to a deleted node must also be deleted

- Graph $G$
- Subgraph $G'$ of $G$
Spanning trees

- $T = (N', A')$ is a spanning tree of $G = (N, A)$ if
  - $T$ is a subgraph of $G$ with $N' = N$ and $T$ is a tree

Graph $G$  

Spanning tree of $G$
Spanning trees

- Spanning trees are useful for disseminating and collecting control information in networks; they are sometimes useful for routing.

- **To disseminate data from Node n:**
  - Node n broadcasts data on all adjacent tree arcs
  - Other nodes relay data on other adjacent tree arcs

- **To collect data at node n:**
  - All leaves of tree (other than n) send data
  - Other nodes (other than n) wait to receive data on all but one adjacent arc, and then send received plus local data on remaining arc
General construction of a spanning tree

• Algorithm to construct a spanning tree for a connected graph $G = (N,A)$:

1) Select any node $n$ in $N$; $N' = \{n\}$; $A' = \{\}$

2) If $N' = N$, then stop ($T=(N',A')$ is a spanning tree)

3) Choose $(i,j) \in A$, $i \in N'$, $j \notin N'$

   $N' := N' \cup \{j\}$; $A' := A' \cup \{(i,j)\}$; go to step 2

• Connectedness of $G$ assures that an arc can be chosen in step 3 as long as $N' \neq N$

• Is spanning tree unique?

• What makes for a good spanning tree?
Minimum Weight Spanning Tree (MST)

- Generic MST algorithm steps:
  - Given a collection of subtrees of an MST (called fragments) add a minimum weight outgoing edge to some fragment

- Prim-Dijkstra: Start with an arbitrary single node as a fragment
  - Add minimum weight outgoing edge

- Kruskal: Start with each node as a fragment;
  - Add the minimum weight outgoing edge, minimized over all fragments
Prim-Dijkstra Algorithm

Step 1

Step 3

Step 4

Step 5
• Suppose the arcs of weight 1 and 3 are a fragment
  – Consider any spanning tree using those arcs and the arc of weight 4, say, which is an outgoing arc from the fragment.
  – Suppose that spanning tree does not use the arc of weight 2.
  – Removing the arc of weight 4 and adding the arc of weight 2 yields another tree of smaller weight.
  – Thus an outgoing arc of min weight from fragment must be in MST.