16.30 Learning Objectives and Practice Problems - - Lectures 16 through 20

IV. Lectures 16-20

IVA: Sampling, Aliasing, and Reconstruction  JVV 9.5, Lecture Notes on Shannon
- Understand the mathematical modeling of the sampling process in computer-controlled systems, and the effect of sampling on the frequency spectrum of a signal.
- Understand the significance of the half-sample (aka Nyquist) frequency, and be able to calculate the aliased frequencies associated with an under-sampled signal.
- Be able to derive and use the transfer function for a zero-order hold, especially in the context of modeling computer-controlled systems.
- Learn how to choose the sample rate for a computer controlled system, and avoid pitfalls associated with under-sampling, aliased noise, and frequency warping.
- Be aware of Shannon’s sampling theorem, and its implications in signal theory.

IVB: Approximating Transfer Functions in Discrete Time & the z operator  JVV 9.6, 10.3 Lecture notes
- Be able to convert transfer functions to discrete-time approximations using the trapezoidal rule, or equivalently Tustin’s transform.
- Learn to manipulate the z-operator into transfer functions, and how to convert between z-domain transfer functions and difference equations, for implementation in a computer.
- Understand the relationship between z-domain poles and time-response behavior of difference equations.

IVC Computer-controlled systems (Lecture Notes)
- Understand the components of a computer-controlled system (sampler, difference equation, zero-order hold) and how to model them.
- Be able to implement a continuous-domain compensator design in a digital computer, using appropriate sample rates, approximations, and verification analysis.
IVA : Sampling, Aliasing, and Reconstruction  JVV 9.5, Lecture Notes on Shannon

- Understand the mathematical modeling of the sampling process in computer-controlled systems, and the effect of sampling on the frequency spectrum of a signal.

  Sketch the spectrum of a sine wave of frequency 20Hz, after it has been ‘mathematically sampled’ (that is, multiplied by an impulse train) at a frequency of 100 Hz. Use the approach in section 9.5 of JVV

- Understand the significance of the half-sample (aka Nyquist) frequency, and be able to calculate the aliased frequencies associated with an under-sampled signal.

HW4, problem 2

Suppose you are conducting an experiment that uses a computer to sample wind-tunnel data at 200 Hz. Someone else has set up the test without regard for anti-aliasing filters or appropriate sample rate. In post-processing, you notice a strong periodic signal in the data, whose period is 50 samples. What are the possible frequencies of the signal you are measuring?

- Be able to derive and use the transfer function for a zero-order hold, especially in the context of modeling computer-controlled systems.

HW4, problem 3

Compare the transfer function of a ZOH with sample rate T to a pure time-delay of duration T/2. What are the similarities and differences? Is there a way to reconstruct a discrete-time signal that does not introduce as much delay as a ZOH?

Using the spectrum of a 20Hz sine wave sampled at 100Hz derived in the problem, at the top of this page, sketch the spectrum of the signal after it is reconstructed by a ZOH. Sketch the time response as well, and comment on the relationship between these two sketches.

- Learn how to choose the sample rate for a computer controlled system, and avoid pitfalls associated with under-sampling, aliased noise, and frequency warping.

Problems 9.1 – 9.3

Can an anti-aliasing filter be implemented within the computer that is being used for control? Why or why not? Assume only one sampler is available, with a fixed sample rate, and that noise exists above the half-sample frequency that you would like to avoid seeing as an aliased signal.

- Be aware of Shannon’s sampling theorem, and it’s implications in signal theory.
**IVB: Approximating Transfer Functions in Discrete Time & the z operator**

- Be able to convert transfer functions to discrete-time approximations using the trapezoidal rule, or equivalently Tustin’s transform.

  **Problems 9.4 and 9.5, part (a) only**

  Problems 9.10, 9.11, and 9.14

- Learn to manipulate the z-operator into transfer functions, and how to convert between z-domain transfer functions and difference equations, for implementation in a computer.

  **Problems 9.22, 10.4, 10.6, 10.7**

- Understand the relationship between z-domain poles and time-response behavior of difference equations, including FIR and IIR transfer functions.

  **Problems 10.10, 10.11, 10.14**

  (a) Write the difference equations for the following two transfer functions. Implement these equations in Matlab, then plot and compare the step responses. Plot each response for a total of 25 samples.

\[
D_1(z) = \frac{z - 1}{z^3 - 1.3z^2 + 0.81z - 0.205}
\]

\[
z^9 + 0.3z^8 - 0.42z^7 - 0.584z^6 - 0.3575z^5 - 0.0778z^4
\]

\[
D_2(z) = \frac{+ 0.0687z^3 + 0.791z^2 + 0.0312z^1 - 0.0094}{z^{11}}
\]

(the latter transfer function is actually an FIR filter, which can be seen by deriving the difference equation)

(b) Plot and compare the frequency responses of these two difference equations, if they were implemented in a digital computer with a sampler and a ZOH.

**IVC Computer-controlled systems**

- Understand the components of a computer-controlled system (sampler, difference equation, zero-order hold) and how to model them.

- Be able to implement a continuous-domain compensator design in a digital computer, using appropriate sample rates, approximations, and verification analysis.

  **Problems 10.30(a), 10.31(a), and 10.32**

  Design a digital filters which will have similar behavior to the following:

  (a) A lag compensator with \( T1/T2 = 10 \), \( 1/T2 = 10 \) rad/sec

  (b) A lead compensator with \( T1/T2 = 15 \), \( w_{mean} = 1 \) rad/sec
(c) A notch filter with a center frequency of 10 rad/sec

In each case, choose a suitable sample frequency for implementation of your filter, and plot the continuous time and the computer-implementation version of each. Your computer-implementation version should contain a sampler and ZOH as well as the transfer function you have created.

Implement the compensator in the solution for Homework 3, Problem 4, as a digital system. Your solution should include:

(a) An description of how you chose the sample rate
(b) The discrete-time transfer function of your compensator
(c) A Bode plot of both the continuous-time and the discrete-time implementation of the controller
(d) Comments on whether there are any problems with your implementation.