16.30 Learning Objectives and Practice Problems - - Lectures 26 through 32

VI. Lectures 26-32

**VIA: Full State Feedback** (JVV 12.1-12.3, 12.7)

- Understand the implications of full state feedback (FSF) in the context of aerospace systems – that is, that proportional and derivative feedback of every system mode (eigenvalue) is imbedded in a FSF control law, since it proportionally feeds back the entire state (i.e \( u = -Kx \)). By analogy to PD control of a 2\textsuperscript{nd} order system, (and by constructive proof provided in class an in the text) complete control of the eigenvalue positions is possible.

- Be able to place the poles of a given system using the tools of FSF, including transformation to control canonical form (aka companion form) and/or application of Ackermann’s formula.

- Have introductory knowledge of the options available for pole placement, sufficient to be able to look up the classically-motivate pole locations and use them in a control problem.

- Understand and be able to apply the basics of controllability analysis, in order to determine whether the conditions for successful FSF-based pole placement are met.

- Be able to implement a full state feedback in a command-following situation, by properly choosing the feedforward gain matrix to be applied to the commanded (reference) outputs.

**VIB: Linear Quadratic Regulator** (JVV 12.4)

- Understand how to formulate the linear LQR problem, and to iteratively manipulate the “Q & R” weighting matrices to achieve desired transient performance properties. Know about the requirements and pitfalls associated with choosing Q & R, the common forms these matrices take, and why these forms are useful and are adequate to meet the criteria of positive semi-definiteness (for Q) and positive definiteness (for R).

- Be familiar with the elements of the derivation of the Ricatti equation, and how it’s solution is used to formulate an LQR feedback law. Be able to manipulate matrices in the context of a matrix Lyapunov equation.

**VIC: Observers and Observer-Based Compensation** (JVV 12.8)

- Learn to formulate an observer that estimates the states of a dynamic system based on measured outputs.

- Learn to analyze the closed-loop properties of a state observer combined with a full-state feedback law (\( u = -Kx \)).

- Understand the separation principal and it’s significance for observer-based control, and the relationship to LQG control.

- Be able to write the equations for, and analyze the input-output properties of, the compensator associated with an observer-based control system.
VIA : Full State Feedback (JVV 11.7, 12.1-12.3, 12.7, class notes.)

- Understand the implications of full state feedback (FSF) in the context of aerospace systems – that is, that proportional and derivative feedback of every system mode (eigenvalue) is imbedded in a FSF control law, since it proportionally feeds back the entire state (i.e \( u = -Kx \)). By analogy to PD control of a 2\textsuperscript{nd} order system, (and by constructive proof provided in class an in the text) complete control of the eigenvalue positions is possible.

- Be able to place the poles of a given system using the tools of FSF, including transformation to control canonical form (aka companion form) and/or application of Ackermann’s formula.

JVV 12.1 – 12.15

- Have introductory knowledge of the options available for pole placement, sufficient to be able to look up the classically-motivate pole locations and use them in a control problem.

Using the 6-state model of the Quanser dynamics from Homework 5, problem 4, design a full state feedback control law to place the poles as described in Table 12.2.1. Use the approach described in remark #2 to convert the system to a single-input system, and then convert the dynamics to control canonical (companion) form using the method described in class. Make sure your choice of alpha in remark #2 yields a controllable system.

- Understand and be able to apply the basics of controllability analysis, in order to determine whether the conditions for successful FSF-based pole placement are met.

JVV 11.35-11.37, also the problem above requires an understanding of controllability.

- Be able to implement a full state feedback in a command-following situation, by properly choosing the feedforward gain matrix to be applied to the commanded (reference) outputs.

Using the procedure described in class, compute the feedforward matrix required to implement command following of theta and psi commands for the Quanser example above. Comment on the form of the feedforward matrix.

Perform the control design specified in problem 12.10. Noting that the second state in this system is the derivative of the first state, draw the block diagram, and write the equations for, a command following system for the state \( x_1 \). You can use the procedure described in class, or simply figure out the correct approach. What is the steady state error associated with this control system?

Now, add an integrator, with an as yet unspecified gain, to the command structure that you created above. Write the resulting state-space representation of the system augmented with integrator if all the feedback gains are set to zero. Finally, find the values for all the gains for this system (including the integrator gains) based on full-state feedback of the system augmented with the integrator.
**VIB: Linear Quadratic Regulator (JVV 12.4)**

- Understand how to formulate the linear LQR problem, and to iteratively manipulate the “Q & R” weighting matrices to achieve desired transient performance properties. Know about the requirements and pitfalls associated with choosing Q & R, the common forms these matrices take, and why these forms are useful and are adequate to meet the criteria of positive semi-definiteness (for Q) and positive definiteness (for R).

**JVV 12.16-12.20**

- Be familiar with the elements of the derivation of the Ricatti equation, and how it’s solution is used to formulate an LQR feedback law. Be able to manipulate matrices in the context of a matrix Lyapunov equation.

Show that if P is the solution of the Lyapunov equation $F^T P + PF = -S$, where P is symmetric and positive definite and $\dot{x} = Fx$ represents the system dynamics, then

$$\frac{d}{dt} (x^T P x) = -x^T S x$$

This proves the primary fact needed in the discussion of optimal control in JVV section 12.4. The proof is easy – just pre-multiply both sides of the Lyapunov equation by $x^T$, post-multiply both sides by x, and use the fact that $\dot{x} = Fx$.

Solve the Lyapunov equation for a simple second-order system.

**VIC: Observers and Observer-Based Compensation (JVV 12.8)**

- Learn to formulate an observer that estimates the states of a dynamic system based on measured outputs.

12.41 – 12.45

- Learn to analyze the closed-loop properties of a state observer combined with a full-state feedback law ($u = -Kx$).

- Understand the separation principal, it’s significance for observer-based control, and the relationship to LQG control.

- Be able to write the equations for, and analyze the input-output properties of, the compensator associated with an observer-based control system.

Design an observer-based compensator based on your full-state feedback solution (or mine) to Homework 7, problem 7. Assume that you can measure the displacements, but not the velocity states of the Quanser. Place the poles of your observer at 2 to 3 times the frequency of the closed-loop poles of the full-state
feedback solution. Formulate the closed-loop state space matrices for this problem, and verify in Matlab that the separation principal is satisfied.