a) \( s = -a \pm j \)

\[ \omega_d = 1 \text{ rad/sec} \]

Sampling theorem: \( \omega_s > 2\omega_d \)

\[ \omega_s > 2 \text{ rad/sec} \]

b) Sample at six times per cycle of transient, or \( \omega_d \)

\[ \omega_s = 6\omega_d \]

\[ \omega_s = 6 \text{ rad/sec} \]

c) Range of values of \( a \) such that \( \omega_s = 6 \text{ rad/sec} \)

will undersample transient

\[ \omega_d = \omega_n \sqrt{1-s^2} = 1 \]

\[ s = \frac{a}{\sqrt{1+a^2}} \]

\[ \omega_n = \sqrt{1+a^2} \]

from JVV 274, pick \( \frac{a}{\sqrt{1+a^2}} \) to be \( t \) of plant time constant (ie \( \frac{1}{a} \))

\[ \omega_s < 10a \]

\[ a > \frac{\omega_s}{10} \]

\[ a > 0.6 \text{ rad/sec} \]

Figure on next page shows changes of step response with increasing \( a \).
Figure 1: The effect of increasing $a$ on the step response of a system with poles at $s = -a \pm j$
Figure 2: (a) Original Signal Spectrum- 120 Hz. (b) Sampled Signal Spectrum- $\omega_s = 100$ Hz. Aliased Frequency at 20 Hz.

Figure 3: Original Sine Wave vs. Undersampled Sine Wave with Aliasing. The period of the aliased signal is about 0.05 sec, which corresponds to a frequency of 20 Hz.
%Code to Generate Bode of ZOH
low_freq = 1; %frequency range
high_freq = 100;
T = 0.1; %sampling time

w = logspace(log10(2*pi*low_freq), log10(2*pi*high_freq), 100);

%TF of ZOH
ZOH = (1-exp(-j.*w.*T))./(j.*w);
mag = 20*log10(abs(ZOH));
phase = (180/pi).*angle(ZOH);

%re-scale frequency from rad/sec to Hz
w = w/(2*pi);

%create Bode plot
figure(1)
subplot(2,1,1),semilogx(w,mag);
xlabel('Frequency (Hz)');
ylabel('Magnitude (dB)');
title('Bode of ZOH');
subplot(2,1,2),semilogx(w,phase);
xlabel('Frequency (Hz)');
ylabel('Phase(deg)');

Figure 4: Bode Plot of Zero Order Hold. Note that the sampling frequency is 10 Hz, which corresponds to the first “dip” in the Bode magnitude plot.
Many students used a Pade approximation to graph the bode of the Zero Order Hold transfer function. Since the problem allowed you to use Matlab, a more accurate and precise graph would have been obtained if a higher-order Pade approximation was used.

Figure 5: ZOH using Pade approximation for $e^{-sT}$. 


Figure 6: ZOH using Pade approximation for $e^{-sT}$. 
% Define sample space (1:25)
samples = linspace(1,25,25);

% Define parameters
a = [0.5 -0.5 1.2 -1.2];
b = [0.0 0.0 -0.6 -0.6];
c = [0.5 1.5 0.4 2.8];

% Unit step input
u = linspace(1,1,25);

% for each set of parameters
for m=1:4
    % Initial conditions are y_{-1} = 0; y_{0} = 0;
    % Setup initial values (Matlab vectors do not have non-positive indices)
    y(1) = c(m)*u(1);
    y(2) = a(m)*y(1) + c(m)*u(2);

    % for the remaining samples,
    % calculate step response using difference equation
    for k=3:25
        y(k) = a(m)*y(k-1) + b(m)*y(k-2) + c(m)*u(k);
    end

    % setup figure
    figure(1);
    % plot step response with discrete samples
    subplot(2,2,m), stem(samples,y);
    xlabel('Sample');
    ylabel('Step Response');
    title(['Step Response for ay_{k-1} + by_{k-2} + cu_k, with a = ',num2str(a(m)),', b = ',num2str(b(m)),', c = ',num2str(c(m))]);
end
Figure 7: Step Responses with various $a$, $b$, and $c$ parameters.