1. Design an observer-based compensator base on your full-state feedback solution (or mine) for Homework 7, problem 7. Assume that you can measure the displacements, but not the velocity states of the Quanser. Perform the following steps:
   a. Write the observer equations, and create the appropriate ‘C’ matrix.

   b. Choose the gain matrix ‘G’ that places the poles of your observer at 2 to 3 times the frequency of the closed-loop poles of the full-state feedback solution. You may use LQR or a pole placement approach.

   c. Formulate the closed-loop state space matrices for this problem, and verify in Matlab that the separation principal is satisfied. For this part of the problem, ignore the servos.

   d. Plot the transient responses computed in Homework 7, and on the same graphs plot the transient responses of your new observer-based compensator. You can do this by simply creating a state-space system for the observer+feedback gain, whose inputs are y and outputs are u, and placing this system at the input to the plant created for Homework 7. Include the servo in this analysis, but remove the limiters.

2. Problem 11.36. Recall that a matrix loses rank if its columns are linear combinations of each other, and that a full rank matrix is invertible.

3. Problem 11.43, plus these related exercises:
   a. Do Problem 11.43 as stated. As I suggested in class, many of the techniques for state space are applicable in the discrete domain. In particular, going from transfer functions to state space representations is straight-forward, and the eigenvalues of the ‘A’ matrix are the poles of the transfer function, and determine stability. The only difference is that discrete-time poles must be inside the unit circle.

   b. Write a short Matlab script which uses the state space difference equations to compute the step response of the system. Plot your result.
4. Perform the control design specified in problem 12.10, and then do the following:
   a. Noting that the second state in this system is the derivative of the first state, draw the block diagram, and write the equations for, a command following system which commands the state x1, but does not command x2. In other words, you want the reference input to the controller to be x1\_desired. You can use the procedure described in class, or simply figure out the correct approach. What is the steady state error associated with this control system? You can analyze this by considering inv(sI-A) as s approaches zero...

5. Show that if P is the solution of the Lyapunov equation \( F^T P + PF = -S \), where P is symmetric and positive definite and \( \dot{x} = Fx \) represents the system dynamics, then:

   \[
   \frac{d}{dt} (x^T P x) = -x^T S x
   \]

   Thus prove the primary fact needed in the discussion of optimal control in JVV section 12.4. The proof is easy – just pre-multiply both sides of the Lyapunov equation by \( x^T \), post-multiply both sides by x, and use the fact that \( \dot{x} = Fx \).