Modeling Uncertainties
There are several ways to model uncertainties:

**Additive**

![Additive diagram]

**Multiplicative**

![Multiplicative diagram]

In both cases, the **nominal plant** occurs when $\Delta = 0$.

The multiplicative error describes the percentage or fractional error; the additive model describes the absolute error.
There are other, more physical models

Example Consider spring mass system with stiffness uncertainty:

\[ u = f \]

\[ m \]

\[ k + \Delta k \]

\[ g = y \]

States:
\[ x_1 = \dot{g} \]
\[ x_2 = g \]

State equation:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix}
= \begin{bmatrix}
0 & 1 \\
\frac{-k + \Delta k}{m} & -d/m
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
+ \begin{bmatrix}
0 \\
1/m
\end{bmatrix}
u
\]

\[
y = \begin{bmatrix}
1 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
\]

\[
\dot{x} = \begin{bmatrix}
0 & 1 \\
-\frac{k}{m} & -d/m
\end{bmatrix}x
+ \begin{bmatrix}
0 \\
-1/m
\end{bmatrix} \Delta k
+ \begin{bmatrix}
0 \\
1/m
\end{bmatrix}u
\]

This looks like feedback!

\[
\begin{bmatrix}
0 & 1 \\
-\frac{1}{m} & -\frac{1}{m}
\end{bmatrix} \dot{x} + \begin{bmatrix}
0 \\
-\frac{1}{m}
\end{bmatrix} p + \begin{bmatrix}
0 \\
\frac{1}{m}
\end{bmatrix} u
\]

A

Bp

\[
q = \begin{bmatrix} 1 & 0 \end{bmatrix} x
\]

cq

\[
y = \begin{bmatrix} 1 & 0 \end{bmatrix} x
\]

\[
p = \Delta x - q
\]

Very often, uncertainty in a control system looks like feedback — plant is a "linear fractional transformation".

This is almost the "standard model"
Additive model:

![Additive Model Diagram]

Multiplicative model:

![Multiplicative Model Diagram]

So almost every uncertainty model is a LFT!
Stability: robustness. Is system stable for all $\Delta \in \Delta$?

Performance: Robustness. Does system meet requirements for all $\Delta \in \Delta$?
To check for stability, break the loop at the Δ:

Example: Spring mass system,

\[ k = 1 \quad d = 0 \quad m = 1 \]

Feedback:

\[ u = -0.2 \quad y = -0.2x_2 \]

\[ \Rightarrow \dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & -0.2 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u \quad \text{for} \quad (1, 0) x \]

\[ \text{Transfer function} \]

\[ \frac{y(s)}{u(s)} = \frac{-1}{p \quad \frac{s^2 + 0.2s + 1}{p}} \]

Bode plot next page-

System is stable for all Δ such that

\[ |\Delta(j\omega)| < 0.2042 \]
But, is stable for all real
\( \Delta k \) such that

\[-1 < \Delta k < \infty\]

So treating \( \Delta k \) as complex is conservative

- It is much easier to treat complex uncertainties

- Remember that this often leads to conservatism.