16.31 Homework 3

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Issued: October 2, 2006
Due: October 11, 2006

Note: Although due October 11, please try to complete by October 6, as there will be another problem set issued then.

1. Given the plant \( G(s) = \frac{1}{s^2} \), design a lead compensator so that the dominant poles are located at \(-2 \pm 2j\).

2. Determine the required compensation for the system

\[
G(s) = \frac{K}{(s + 8)(s + 14)(s + 20)}
\]

to meet the following specifications:

(a) Overshoot \( \leq 5\% \)
(b) 10-90\% rise time \( t_r \leq 150\) msec

Simulate the response of this closed-loop system to a step response. Comment on the steady-state error. You should find that it is quite large.

Determine what modifications you would need to make to this controller so that the system also has

(c) \( K_p > 6 \)

thereby reducing the steady state error. Simulate the response of this new closed-loop system and confirm that all the specifications are met.

3. Develop a state space model for the transfer function (not in modal / diagonal form).

\[
G_1(s) = \frac{(s + 1)(s + 2)}{(s + 3)(s + 4)}
\]

Discuss what state vector you chose and why. Develop a “modal” state space model for this transfer function as well. Confirm that both models yield the same transfer function when you compute

\[
\hat{G}(s) = C(sI - A)^{-1}B + D
\]
4. A set of state-space equations is given by:

\[
\begin{align*}
\dot{x}_1 &= x_1 (u - \beta x_2) \\
\dot{x}_2 &= x_2 (-\alpha + \beta x_1)
\end{align*}
\]

where \( u \) is the input and \( \alpha \) and \( \beta \) are positive constants.

(a) Is this system linear or nonlinear, time-varying or time-invariant?

(b) Determine the equilibrium points for this system (constant operating points), assuming a constant input \( u = 1 \).

(c) Near the positive equilibrium point from (b), find a linearized state-space model of the system.