I. Equation of State:

\[ pv = RT \quad \text{or} \quad p = \rho RT \quad \text{for a thermally perfect gas} \]

II. Expressions for Work:

A. Work for a simple compressible substance

\[ W = \int_{V_i}^{V_f} p_{ext} dV \]

B. Work for a simple compressible substance undergoing a quasi-static process

\[ W = \int_{V_i}^{V_f} p dV \]

C. Work for an isothermal, quasi-static process of a simple compressible substance

\[ W = mRT \cdot \ln \left( \frac{V_f}{V_i} \right) = mRT \cdot \ln \left( \frac{p_1}{p_2} \right) \]

D. Work for an isobaric quasi-static process of a simple compressible substance

\[ W = p(V_f - V_i) \]

E. Work for a quasi-static adiabatic process

\[ W = -(U_f - U_i) \]

F. Work for quasi-static adiabatic process of an ideal gas

\[ W = -mc_v(T_f - T_i) \]

III. Forms of the First Law of Thermodynamics

A. Most general forms

\[ \Delta E = Q - W, \quad \Delta e = q - w, \quad dE = \delta Q - \delta W, \quad \text{and} \quad de = \delta q - \delta w \]

B. Neglecting changes in kinetic and potential energy

\[ \Delta U = Q - W \quad \Delta u = q - w, \quad dU = \delta Q - \delta W, \quad \text{and} \quad du = \delta q - \delta w \]

C. Neglecting changes in kinetic and potential energy, in terms of enthalpy
\[ H = U + pV \]  
therefore  
\[ dH = dU + pdV + Vdp \]

so  
\[ dH = \delta Q - \delta W + pdV + Vdp \]

or  
\[ dh = \delta q - \delta w + pdv + vdp \]

D. For quasi-static processes where changes in kinetic and potential energy are not important.

\[ dU = \delta Q - pdV \quad \text{or} \quad du = \delta q - pdv \]

\[ dH = \delta Q + Vdp \quad \text{or} \quad dh = \delta q + vdp \]

E. For quasi-static processes of an ideal gas where changes in kinetic and potential energy are not important.

\[ mc_vdT = \delta Q - pdV \quad \text{or} \quad c_vdT = \delta q - pdv \]

\[ mc_pdT = \delta Q + Vdp \quad \text{or} \quad c_pdT = \delta q + vdp \]

IV. The First Law of Thermodynamics as a Rate Equation

A. Most general form

\[
\frac{dE_{c.v.}}{dt} = \dot{Q}_{c.v.} - \dot{W}_{c.v.} + \dot{m}_{\text{in}} e_{\text{in}} - \dot{m}_{\text{out}} e_{\text{out}}
\]

\[
\begin{array}{l}
\text{(rate of change)} \\
\text{(of energy in c.v.)} \\
\text{(rate of heat)} \\
\text{(added to c.v.)} \\
\text{(rate of work)} \\
\text{(done)} \\
\text{(rate of energy)} \\
\text{(flow in to c.v.)} \\
\text{(flow out of c.v.)}
\end{array}
\]

B. For a steady flow process

\[
\frac{d}{dt} = 0 \quad \text{and} \quad \dot{m}_{\text{in}} = \dot{m}_{\text{out}} = \dot{m}
\]

\[
\dot{Q}_{c.v.} - \dot{W}_{c.v.} = \dot{m}(e_{\text{out}} - e_{\text{in}})
\]

or

\[
\dot{Q}_{c.v.} - \dot{W}_{c.v.} = \dot{m}\left[(IE + KE + PE)_{\text{out}} - (IE + KE + PE)_{\text{in}}\right]
\]
C. For a steady flow process neglecting changes in potential energy

\[
\dot{Q}_{c.r.} - \dot{W}_{c.r.} = m \left[ \left( u + \frac{c^2}{2} \right)_{out} - \left( u + \frac{c^2}{2} \right)_{in} \right] \\
\text{or}
q_{1-2} - w_{1-2} = u_2 - u_1 + \frac{c_2^2}{2} - \frac{c_1^2}{2}
\]

written in terms of external or shaft work

\[
q_{1-2} - w_{st-2} = (u_2 + p_2 v_2) - (u_1 + p_1 v_1) + \frac{c_2^2}{2} - \frac{c_1^2}{2}
\]

or in terms of shaft work and enthalpy

\[
q_{1-2} - w_{st-2} = h_2 - h_1 + \frac{c_2^2}{2} - \frac{c_1^2}{2}
\]

D. Steady flow energy equation for an ideal gas

\[
q_{1-2} - w_{st-2} = \left( c_p T_2 + \frac{c_2^2}{2} \right) - \left( c_p T_1 + \frac{c_1^2}{2} \right)
\]

E. Steady flow energy equation for an ideal gas for an adiabatic process with no shaft work

\[
c_p T_2 + \frac{c_2^2}{2} = c_p T_1 + \frac{c_1^2}{2}
\]

The quantity that is conserved is called the stagnation temperature.

\[
T_r = T + \frac{c^2}{2c_p} \quad \left( \text{or} \quad \frac{T_r}{T} = 1 + \frac{\gamma - 1}{2} M^2 \quad \text{usin g} \quad a = \sqrt{\gamma RT} \right)
\]

It is also convenient to define the stagnation enthalpy, \( h_r \)

\[
h_r = c_p T + \frac{c^2}{2}
\]

so we can rewrite the Steady Flow Energy Equation in a convenient form as

\[
q_{1-2} - w_{st-2} = h_{r2} - h_{r1}
\]
F. Steady flow energy equation for an ideal gas for a quasi-static adiabatic process with no shaft work

\[ \frac{p_T}{p} = \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma - 1}} \]

G. For a uniform state, uniform flow process from time \( t_1 \) to time \( t_2 \)

\[
\left[ m_2 \left( u + \frac{c^2}{2} + gZ \right) \right]_{t_2} - m_1 \left( u + \frac{c^2}{2} + gZ \right)_{t_1} + \sum m_{\text{out}} \left( h + \frac{c^2}{2} + gZ \right)_{\text{out}} - \sum m_{\text{in}} \left( h + \frac{c^2}{2} + gZ \right)_{\text{in}} = Q_{C,V} - W_{S,C,V}.
\]

H. For a steady process in terms of molar flow rates and enthalpy per mole (neglecting changes in kinetic and potential energy)

\[ \sum \dot{n}_{\text{out}} \bar{h}_{\text{out}} - \sum \dot{n}_{\text{in}} \bar{h}_{\text{in}} = \dot{Q} - W_s. \]

V. Other relationships

A. Relationship between properties for quasi-static, adiabatic processes for thermally perfect gases

\[ pv^\gamma = \text{constant} \]

\[ \frac{p_2}{p_1} = \left( \frac{T_2}{T_1} \right)^{\gamma/\gamma - 1} \quad \text{and} \quad \frac{T_2}{T_1} = \left( \frac{v_1}{v_2} \right)^{\gamma - 1} \quad \text{and} \quad \frac{p_2}{p_1} = \left( \frac{v_1}{v_2} \right)^\gamma \]

B. Thermal efficiency of a cycle

\[ \eta = \frac{\text{net work}}{\text{heat input}} = \frac{\Delta w}{q_{\text{comb}}} \]

C. Quality, \( x \), for two-phase systems

\[ x = \frac{\text{mass vapor}}{\text{total mass}} = \frac{v - v_f}{v_{fg}} \quad \text{or} \quad v = (1 - x)v_f + xv_g \]

D. Entropy

\[ ds = c_v \frac{dT}{T} + R \frac{dv}{v} \]
For the case of a thermally perfect gas then

\[ s - s_0 = \int_{T_0}^{T} c_v \frac{dT}{T} + R \ln \left( \frac{v}{v_0} \right) \]

or in situations with \( c_v = \text{constant} \)

\[ s - s_0 = c_v \ln \left( \frac{T}{T_0} \right) + R \ln \left( \frac{v}{v_0} \right) \]

So for the case of a thermally perfect gas then

\[ s - s_0 = \int_{T_0}^{T} c_p \frac{dT}{T} - R \ln \left( \frac{p}{p_0} \right) \]

or in situations with \( c_v = \text{constant} \)

\[ s - s_0 = c_p \ln \left( \frac{T}{T_0} \right) - R \ln \left( \frac{p}{p_0} \right) \]

**VI. Nomenclature**

- \( a \): speed of sound (m/s)
- \( c \): velocity (m/s)
- \( c_p \): specific heat at constant pressure (J/kg-K)
- \( c_v \): specific heat at constant volume (J/kg-K)
- \( e \): energy (J/kg)
- \( E \): energy (J)
- \( h \): enthalpy (J/kg)
- \( h_T \): total or stagnation enthalpy (J/kg)
- \( H \): enthalpy (J)
- \( m \): mass (kg)
- \( n \): number of moles (moles)
- \( p \): pressure (kPa)
- \( p_T \): total or stagnation pressure (kPa)
- \( q \): heat (J/kg)
- \( Q \): heat (J)
- \( R \): gas constant (J/kg-K)
- \( s \): entropy (J/K)
- \( S \): entropy (J/kg-K)
- \( t \): time (s)
- \( T \): temperature (K)
- \( T_T \): total or stagnation temperature (K)
- \( u \): internal energy (J/kg)
- \( U \): internal energy (J)
- \( v \): specific volume (m³/kg)
- \( V \): volume (m³)
\( w \) work (J/kg)
\( w_s \) shaft or external work (J/kg)
\( W \) work (J)
\( X \) quality
\( \gamma \) ratio of specific heats, \( c_p/c_v \)
\( \eta \) thermal efficiency
\( \rho \) density (kg/m\(^3\))