Recitation Problem 1 (Unified Thermodynamics)

Air (assume it behaves as a thermally perfect gas) is contained in a vertical cylinder that is fitted with a frictionless piston and a set of stops, as shown below. The piston cross-sectional area is 0.5m² and the air inside is initially at 200kPa and 800K. The air is then slowly cooled as a result of heat transfer to the surroundings. (LO#1, LO#4, LO#5)

a) Sketch these two processes on p-V and T-v diagrams

b) What is the temperature of the air inside the cylinder when the piston reaches the stops?

c) After the piston hits the stops, the cooling is continued until the temperature reaches 300K. What is the pressure at this state?

d) How much work is done by the system in the first cooling process?

e) How much work is done by the system in the second cooling process?
b) Given two independent properties for initial condition, fully defines the state of the system.

\[ P = 200 \text{kPa}, \ T = 800 \text{K} \]

\[ P = gRT \quad \Rightarrow \quad \rho = \frac{0.87 \text{ kg/m}^3}{R = 287 \text{ J/kg·K}} \quad \Rightarrow \quad V = 1 \text{m}^3 \quad \text{(from sketch)} \]

Cooled at constant pressure to \( \frac{1}{2} \) volume.

\[ \rho = (0.87) \frac{1}{2} \text{ kg/m}^3 = 1.74 \text{ kg/m}^3 \]

Now use ideal gas law to get temp.

\[ T = \frac{200 \text{kPa}}{1.74 \text{kg/m}^3 \cdot 287 \text{J/kg·K}} = 400 \text{K} \]

\[ T = 400 \text{K} \]

C) Now constant volume cooling.

\[ \rho = 1.74 \text{ kg/m}^3 = \text{constant} \]

\[ T = 300 \text{K} \quad \Rightarrow \quad P = 1.74 \cdot 287 \cdot 300 = 150 \text{kPa} \]
Recitation Problem 1 (Unified Thermodynamics)

a) **Work done in first process.**

Assume it is a quasi-static process.

\[ P = \text{const.} \quad \text{so} \quad W = P \Delta V \quad \left( \frac{J}{kJ} \right) \]

or \[ W = p \Delta V \quad (J) \]

\[ W = 200 \text{ kPa} \left( V_{\text{final}} - V_{\text{initial}} \right) \]

\[ W = -200 \text{ kJ} \]

\[ = 200 \text{ kPa} \left( -1 \text{ m}^3 \right) \]

b) **Work done by system in second cooling process is zero.**
Recitation Problem 2 (Unified Thermodynamics)

Consider a thermodynamic system containing air at $v_1 = 1 \text{ m}^3/\text{kg}$, $p_1 = 100 \text{ kPa}$. The system is compressed to $0.5 \text{ m}^3/\text{kg}$ via anyone of three quasi-static processes: isobaric, isothermal, or adiabatic. Assume that $c_p = 1.0035 \text{ kJ/kg-K}$, $c_v = 0.7165 \text{ kJ/kg-K}$, and $R = 0.287 \text{ kJ/kg-K}$. (LO#4, LO#5)

a) Sketch all three processes on the same p-v diagram.

b) For each process determine the pressure and temperature at the final state.

c) For each process determine the work done by the system and the heat transferred to the system.
Recitation Problem 2 (Unified Thermodynamics)

a) $P$

\begin{itemize}
  \item[(b)] INITIAL CONDITIONS
    \begin{align*}
      V_i &= 1 \text{ m}^3/\text{kg}, \quad P_i = 100 \text{kPa} \quad \therefore \quad T_i &= 348.4 \text{ K} \\
      \text{by ideal gas law}
    \end{align*}

    \begin{enumerate}
      \item ISOBARIC
        \begin{align*}
          P &= \text{const.} \\
          P_i &= P_2 = 100 \text{kPa} \\
          \therefore \quad V_2 &= 0.5 \text{ m}^3/\text{kg}
        \end{align*}
      \item ISONTHERMAL
        \begin{align*}
          T &= \text{const} \\
          T_1 &= T_2 = 348.4 \text{ K} \\
          \therefore \quad V_2 &= 0.5 \text{ m}^3/\text{kg}
        \end{align*}
      \item Q-S ADIABATIC
        \begin{align*}
          \frac{P_2}{P_i} &= \left( \frac{T_2}{T_1} \right)^{\gamma - 1} \\
          \therefore \quad T_2 &= 174.2 \text{ K} \quad \text{by ideal gas law}
        \end{align*}
    \end{enumerate}

    \begin{enumerate}
      \item \[
          P_2 = 200 \text{kPa}
        \] \quad \text{by ideal gas law}
    \end{enumerate}

    \begin{align*}
      3) \quad q-s \text{ adiabatic} \\
      \frac{P_2}{P_i} &= \left( \frac{V_1}{V_2} \right)^{\gamma - 1} \\
      \therefore \quad \frac{T_2}{T_1} &= 1.32 \Rightarrow T_2 &= 459.7 \text{ K}
    \end{align*}

    \begin{align*}
      \frac{P_2}{P_i} &= \left( \frac{T_2}{T_1} \right)^{\gamma - 1} \\
      \therefore \quad P_2 &= 264 \text{kPa}
    \end{align*}
Recitation Problem 2 (Unified Thermodynamics)

e) For each process, calculate work done by system $\dot{W}$.

1) Isochoric

\[ \dot{W} = p \Delta V \]

\[ \dot{W} = 100 \text{ kPa} \times (0.5 \text{ m}^3 / \text{ kg} - 1.0 \text{ m}^3 / \text{ kg}) = -50 \text{ kJ/kg} \]

1st law: $\Delta U = q - w$

\[ q = \Delta U + w = C_V(T_2 - T_1) + w \]

\[ q = 716.5(459.7 - 348.4) + (-50,000) \]

\[ q = -175 \text{ kJ/kg} \]

2) Isothermal

\[ \dot{W} = RT \ln \left( \frac{V_2}{V_1} \right) = 287.348.4 \ln (0.5) \]

\[ \dot{W} = -69 \text{ kJ/kg} \]

1st law: $\Delta U = q - w$\n
\[ q = w \]

\[ q = -69 \text{ kJ/kg} \]

3) Adiabatic

\[ \Delta U = q - w \]

\[ q = 0 \]

\[ \dot{W} = -\Delta U = -C_v (T_2 - T_1) = -716.5(459.7 - 348.4) \]

\[ \dot{W} = -80 \text{ kJ/kg} \]

Compressed