Unified Thermodynamics Quiz 2

December 20, 2006

Calculators allowed.
No books or notes allowed.
A list of equations is provided.
Tables of thermodynamic properties are provided.

- Put your ID number on each page of the exam.
- Do all work for each problem on the pages provided.
- Show intermediate results. Partial credit will be given (unless otherwise noted), but only when the intermediate results and explanations are clear.
- Explain your work --- don’t just write equations.
- Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
- Show appropriate units with your final answers.
- Box your final answers.

Exam Scoring

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1) A fully-evacuated \((p = 0, \text{no mass in the tank})\), thermally-insulated, tank is sitting in a large volume of a substance that exists at \(T_{\text{ambient}}\) and \(p_{\text{ambient}}\). A valve is opened such that the substance outside of the tank flows into the tank until the pressure inside the tank is equal to \(p_{\text{ambient}}\).

a) (8%, LO#2, LO#4) If the substance outside of the tank is saturated water vapor, how does the temperature of the substance inside of the tank compare to \(T_{\text{ambient}}\) when pressure equilibrium is reached? Why? Substantiate your answer with reference to appropriate equations, thermodynamic properties and physical behavior of substances.

The first law

\[
\left[ m_1 \left( u + \frac{c^2}{2} + gz \right)_{\text{in}} - m_1 \left( u + \frac{c^2}{2} + gz \right)_{\text{out}} \right] + \sum m_{\text{in}} \left( h + \frac{c^2}{2} + gz \right)_{\text{in}} - \sum m_{\text{out}} \left( h + \frac{c^2}{2} + gz \right)_{\text{out}} = Q_{\text{c}} - W_{\text{s}}
\]

becomes \(u_{\text{tank}} = h_{\text{ambient}}\). Therefore, the temperature in the tank is higher than the ambient temperature (since \(pv > 0\)). If we consider the mass that ends up in the tank as the system, then work is done on the system by the surroundings \((p-v \text{ work})\) such that the system’s internal energy is higher than that of the surroundings. [Consider \(p_{\text{ambient}} = 100\text{kPa}\) then \(T_{\text{ambient}} = 99.62 \degree \text{C}\) and, \(h_{\text{ambient}} = 2675.46 \text{ kJ/kg}\) (from steam tables). So \(u_{\text{tank}} = 2675.46\text{kJ/kg}, \) \(x = 0.54\) and \(T_{\text{tank}} = 211.5 \degree \text{C}\) (from superheated vapor tables).]

b) (8%, LO#2, LO#4) If the substance outside of the tank is water with a quality of 0.5, how does the temperature inside the tank when pressure equilibrium is reached compare to the temperature inside the tank in part (a)? Why? Substantiate your answer with reference to appropriate equations, thermodynamic properties and physical behavior of substances.

The temperature inside the tank is lower than that reached in part (a) since some of the \(p-v\) work goes into changing the phase of the substance. The amount that goes into changing the phase and changing the temperature depends on the conditions. [Consider \(p_{\text{ambient}} = 100\text{kPa}, x = 0.5\), then \(T_{\text{ambient}} = 0.62 \degree \text{C}\) and, \(h_{\text{ambient}} = 1546.45 \text{ kJ/kg}\) (from steam tables). So \(u_{\text{tank}} = 1546.45 \text{kJ/kg}, x = 0.54\) and \(T_{\text{tank}} = 99.62 \degree \text{C}\) (from steam tables). In this case all the energy goes into the phase change and the temperature is the same as the ambient temperature.]
2) A device called a heat exchanger is shown below. The cold side flows water (tables of properties are provided with the exam). The hot side flows air which can be assumed to behave as an ideal gas with $R = 287$ J/kg-K, $c_p = 1003.5$ J/kg-K, and $c_v = 716.5$ J/kg-K.

H$_2$O

10 kg/s

$T=573.15K$

= 300 $^\circ$C

c=60 m/s

$p=200kPa$

Air

25kg/s

$T=1500K$

c=100 m/s

$p=400kPa$

H$_2$O, $x=0.5$

10 kg/s

$T=?$

c=50 m/s

$p=200kPa$

Air

25kg/s

$T=?$

c=75 m/s

$p=100kPa$

---

a) (8%, LO#2) Draw the inlet and outlet states for the hot side and the cold side on two separate p-v diagrams and sketch what the process may look like in between these states (show the states relative to the vapor dome for the cold side).
2) Continued…

b) (5%, LO#1, LO#4) Describe the energy exchange processes in this device in terms of heat, work and various forms of energy.

Both the hot and cold-side flows enter the device with internal and kinetic energy. The temperature of the hot side flow is higher, so there is heat transfer between the two (energy flows from the hot side to the cold side). This heat transfer raises the kinetic and internal energy of the cold side flow and lowers the kinetic and internal energy of the hot side flow. There is a flow of energy out of the device due to both streams exiting (carrying internal and kinetic energy). There is also net flow work done by the streams entering and exiting the device. The balance of all the energy flows into and out of the device is zero since the energy of the device itself does not change with time.

c) (20%, LO#4) What is the temperature at the exit of the hot side flow?

On the cold side enough information is given to fully determine the conditions entering and exiting the device (inlet pressure and quality are given, thus uniquely determining the temperature). This allows the heat transfer to be determined. Then for the hot side, inlet conditions and the heat transfer are known, so temperature, the only unknown exit condition, can be determined.

COLD SIDE inlet: \( c = 50 \text{ m/s}, p = 200 \text{ kPa}, x = 0.5 \), therefore, from the steam tables \( T = 120.23 \text{ °C} = 393.38 \text{K}, v = 0.4434 \text{ m}^3/\text{kg}, u = 1516.98 \text{ kJ/kg}, h = 1605.66 \text{ kJ/kg} \).

COLD SIDE outlet: \( c = 60 \text{ m/s}, p = 200 \text{ kPa}, \) superheated vapor, \( T = 300 \text{ °C} \), therefore from the superheated vapor tables, \( v = 1.31616 \text{ m}^3/\text{kg}, u = 2808.55 \text{ kJ/kg}, h = 3071.79 \text{ kJ/kg} \).

\[
\dot{Q}_{c.v.} - \dot{W}_{c.v.} = \dot{m} \left[ \left( u + \frac{c^2}{2} \right)_{\text{out}} - \left( u + \frac{c^2}{2} \right)_{\text{in}} \right] \rightarrow \dot{Q}_{c.v.} - \dot{W}_{s,c.v.} = \dot{m} \left[ \left( h + \frac{c^2}{2} \right)_{\text{out}} - \left( h + \frac{c^2}{2} \right)_{\text{in}} \right]
\]

No shaft work, mass flow = 10 kg/s

\[
\dot{Q}_{c.v.} = \dot{m} \left[ \left( h + \frac{c^2}{2} \right)_{\text{out}} - \left( h + \frac{c^2}{2} \right)_{\text{in}} \right] = 10 \left[ \left( 3071.79 e03 + \frac{60^2}{2} \right)_{\text{out}} - \left( 1605.66 e03 + \frac{50^2}{2} \right)_{\text{in}} \right]
\]

= 14667 kJ/s (positive, so heat added to the system as expected)
2) Continued…

Now we can use this as the heat extracted from the hot side and determine the unknown conditions at the exit of the hot side.

\[
\dot{Q}_{c,v} = m \left[ h + \frac{c^2}{2} \right]_{\text{out}} - \left[ h + \frac{c^2}{2} \right]_{\text{in}} = m \left[ c_p(T_{\text{out}} - T_{\text{in}}) + \frac{c_{\text{out}}^2}{2} - \frac{c_{\text{in}}^2}{2} \right] = -14667e03 \text{ J/s}
\]

Note change of sign on heat transfer – now negative since removed from system.

Mass flow is 25 kg/s.

HOT SIDE inlet: \( c = 100 \text{ m/s}, \ p = 400 \text{ kPa}, \) air at \( T = 1500 \text{ K} \)

HOT SIDE outlet: \( c = 75 \text{ m/s}, \ p = 100 \text{ kPa}, \) air at \( T = ??? \text{ K} \).

\[
25 \left[ 1003.5(T_{\text{out}} - 1500) + \frac{75^2}{2} - \frac{100^2}{2} \right] = -14667e03 \text{ J/s}
\]

\( T_{\text{out}} = 917.6 \text{ K} \)

d) (8%, LO#1, LO#2) This is a quote from Thermodynamics for Engineers by Wong,©2000 by CRC Press. “No work is done in a heat exchanger.” Do you agree or disagree and why?

Please substantiate your argument with a calculation.

There is no shaft work done, but there is flow work (i.e. the air and the water enter and exit the device with different values of \( pv \)). Note, can only use the ideal gas equation \( (pv = RT) \) for an ideal gas, not for the 2-phase water mixture.

\[
\dot{W}_{\text{flow-coldside}} = \dot{m}p(v_{\text{out}} - v_{\text{in}}) = 10 * 200e03 * (1.31616 - 0.4434) = (1746 \text{ kW})
\]

\[
\dot{W}_{\text{flow-hotside}} = \dot{m}R(T_{\text{out}} - T_{\text{in}}) = 25 * 287(917.6 - 1500) = (-4179 \text{ kW})
\]

So there is net flow work.

e) (8%, LO#5) Is the process in this device reversible or irreversible and why? (Do not do a calculation; answer with a few sentences)

The process is irreversible. It is like putting a hot brick and a cold brick together. Although it would be possible to reverse the process (and transfer energy from the cold side to the hot side) it cannot be done without changing the surroundings.
3) A ramjet is composed of an inlet, a combustor and a nozzle. Hydrogen fuel is injected into the combustion chamber. Assume the ratio of specific heats, $\gamma = 1.4$.

![Ramjet Diagram](http://en.wikipedia.org/wiki/Ramjet)

Image from: http://en.wikipedia.org/wiki/Ramjet

a) **(10%, LO#4)** The ramjet is moving at $M = 5$ through a still atmosphere. The temperature of the atmosphere is 260 K and the pressure is 10 kPa. What are the static and stagnation temperatures and static and stagnation pressures in the frame of reference of the ramjet?

For an adiabatic process with no external work

$$ \frac{T_T}{T} = 1 + \frac{\gamma - 1}{2} M^2 $$

and if also quasi-static, then

$$ \frac{P_T}{p} = \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\gamma / (\gamma - 1)}$$

$T = 260K$, $p = 10$ kPa, therefore, $T_T = 1560$ K and $P_T = 5.29$ MPa.

b) **(10%, LO#4)** If the flow through the engine is decelerated quasi-statically and adiabatically so that at the entrance of the combustor it is $M = 0.5$ relative to the reference frame of the ramjet, what are the temperature and pressure of the flow? What are the stagnation temperature and stagnation pressure?

In frame of reference of the ramjet, $T_T = 1560K$, $P_T = 5.29$ MPa and $M = 0.5$. Therefore $T = 1486$ K and $p = 4.46$ MPa using the equations from part (a).
3) Continued…

c) (15%, LO#4) For each kg/s of air flowing through the engine, 0.02 kg/s of gaseous hydrogen is injected and burned in the combustor. Assume the combustion occurs at constant pressure and the changes in kinetic and potential energy are negligible compared to the changes in enthalpy. If the temperature at the exit of the combustor is 1800K how much heat transfer occurred in the combustion chamber for each kg/s of air flow rate (note there are 3.76 moles of \(N_2\) for each mole of \(O_2\) in air)?

***NOTE: You are expected to write each equation in detail and work the answer out numerically until the point where you would have to look up values for thermodynamic properties in a table. You should proceed symbolically from that point forward. If you have questions about whether you are proceeding correctly, please ask me.***

The fuel-air ratio on a mass basis is 0.02. Must determine what the fuel-air ratio is on a molar basis.

\[
\frac{m_{\text{fuel}}}{m_{\text{air}}} = \frac{n_{\text{fuel}} MW_{\text{fuel}}}{n_{\text{air}} MW_{\text{air}}} \quad \therefore \quad \frac{n_{\text{fuel}}}{n_{\text{air}}} = \frac{m_{\text{fuel}} MW_{\text{air}}}{m_{\text{air}} MW_{\text{fuel}}} = \frac{0.02 (32 + 3.76 \times 28)}{2} = 1.3728
\]

Now set up the chemical equation:

\[
1.3728H_2 + O_2 + 3.76N_2 \rightarrow 1.3728H_2O + 0.3136O_2 + 3.76N_2
\]

Now use the First Law to determine the heat transfer:

\[
\sum n_{\text{out}} \bar{h}_{\text{out}} - \sum n_{\text{in}} \bar{h}_{\text{in}} = \dot{Q} - \dot{W}_s
\]

Assuming no external work, for each mole of air the First Law becomes:

\[
\left(1.3728 * h_{\text{H}_2\text{O}}(T_{\text{out}}) + 0.3136 * h_{\text{O}_2}(T_{\text{out}}) + 3.76 * h_{\text{N}_2}(T_{\text{out}})\right) - \left(1.3728 * h_{\text{H}_2}(T_{\text{in}}) + 1 * h_{\text{O}_2}(T_{\text{in}}) + 3.76 * h_{\text{N}_2}(T_{\text{in}})\right) = \dot{Q} \quad [J/mole]
\]

This is the number of Joules transferred for each mole of air. To determine the number of Joules for each kilogram of air it is necessary to divide by the molecular weight of one mole of air:

\[
\dot{Q}[J/s] = \dot{Q}[J/mole] * \text{moles/s} = \dot{Q}[J/mole] * kg/s * \text{moles/kg}
\]
Unified Engineering Thermo Quiz 2

ID NUMBER______________________

Fall 2006

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