Unit M4.7
The Column and Buckling

Readings:
CDL 9.1 - 9.4
CDL 9.5, 9.6

16.003/004 -- “Unified Engineering”
Department of Aeronautics and Astronautics
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LEARNING OBJECTIVES FOR UNIT M4.7

Through participation in the lectures, recitations, and work associated with Unit M4.7, it is intended that you will be able to........

• ....explain the concepts of stability, instability, and bifurcation, and the issues associated with these

• ....describe the key aspects composing the model of a column and its potential buckling, and identify the associated limitations

• ....apply the basic equations of elasticity to derive the solution for the general case

• ....identify the parameters that characterize column behavior and describe their role
We are now going to consider the behavior of a rod under compressive loads. Such a structural member is called a column. However, we must first become familiar with a particular phenomenon in structural behavior, the….

**Concept of Structural Stability/Instability**

Key item is transition, with increasing load, from a stable mode of deformation (stable equilibrium for all possible [small] displacements/deformations, a restoring force arises) to an unstable mode of deformation resulting in collapse (loss of load-carrying capability)

Thus far we have looked at structural systems in which the stiffness and loading are separate…..
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General: \( k \) = \( x \) = \( F \)

There are, however, systems in which the effective structural stiffness depends on the loading.

Define: effective structural stiffness \( (k) \) is a linear change in restoring force with deflection.

That is: \( \frac{dF}{dx} = k \)
Examples

String (stiffening)

frequency changes with load and frequency is a function of stiffness

Ruler/pointer (destiffening)

easier to push in $x_1$, the more it deflects in $u_3$

--> From these concepts we can define a static (versus dynamic such as flutter -- window blinds) instability as:

“A system becomes unstable when a negative stiffness overcomes the natural stiffness of the structural system”

that is there is a

“loss of natural stiffness due to applied loads”
Let’s make a simple model to consider such phenomenon….

Consider a rigid rod with torsional spring with a load along the rod and perpendicular to the rod.

Figure M4.7-1  Rigid rod attached to wall with torsional spring

Restrict to small deflections (angles) such that $\sin \theta \approx \theta$
Use moment equilibrium:

\[ \sum M(\text{origin}) = 0 \quad \Rightarrow \quad -P_1L - P_2L\sin\theta + k_T\theta = 0 \quad \Rightarrow \quad -M_A = M_A \]
get: \( \left( \frac{k_T - P_2L}{L} \right) \theta = P_1 \)

effective torsional stiffness

i.e., \( k_{\text{eff}} \theta = P \)

Note: load affects stiffness: as \( P_2 \) increases, \( k_{\text{eff}} \) decreases

*Important value*: if \( P_2 L = k_T \)

\[ \Rightarrow k_{\text{eff}} = 0 \]

Point of “static instability” or “buckling”

\[ P_2 = \frac{k_T}{L} \]

Note terminology: \textit{eigenvalue} = value of load for static instability

\textit{eigenvector} = displacement shape/mode of structure (we will revisit these terms)

Also look at \( P_2 \) acting alone and “perturb” the system (give it a \( \Delta \) deflection; in this case \( \Delta \theta \))

\textit{stable}: system returns to its condition

\textit{unstable}: system moves away from condition
Figure M4.7-3  Rod with torsional spring perturbed from stable point

Sum moments to see direction of motion

$$\sum M \Rightarrow -P_2 L \sin \Delta \theta + k_T \Delta \theta \alpha \dot{\theta}$$
(proportional to change in $\theta$)

$$\Rightarrow \left( k_T - P_2 L \right) \Delta \theta \alpha \dot{\theta}$$

Note: $-\dot{\theta}$ is CCW (restoring)
$+\dot{\theta}$ is CW (unstable)
So: if \( k_T > P_2L \) \( \Rightarrow \) stable and also get \( \theta = 0 \)

if \( P_2L \geq k_T \) \( \Rightarrow \) unstable and also get \( \theta = \infty \)

critical point: \( P_2 = \frac{k_T}{L} \)

\( \Rightarrow \) spring cannot provide a sufficient restoring force

--> so for \( P_2 \) acting alone:

*Figure M4.7-4*  Response of rod with torsional spring to compressive load along rod

![Diagram](image-url)
ABC - Equilibrium path, but not stable
ABD - Equilibrium path, deflection grows unbounded ("bifurcation") (B is bifurcation point, for simple model, ...2 possible equilibrium paths)

**Note:** If $P_2$ is negative (i.e., upward), stiffness increases

--> contrast to deflection for $P_1$ alone

*Figure M4.7-5*  Response of rod with torsional spring to load perpendicular to rod

--> Now put on some given $P_1$ and then add $P_2
Figure M4.7-6  Response of rod with torsional spring to loads along and perpendicular to rod

Note 1: If $P_2$ and $P_1$ removed prior to instability, spring brings bar back to original configuration (as structural stiffnesses do for various configurations)

Note 2: Bifurcation is a mathematical concept. The manifestations in actual systems are altered due to physical realities/imperfections. Sometimes these differences can be very important.
We’ll touch on these later, but let’s first develop the basic model and thus look at the….

**Definition/Model of a Column**

(Note: we include stiffness of continuous structure here. Will need to think about what is relevant structural stiffness here.)

a) **Geometry** - The basic geometry does not change from a rod/beam

*Figure M4.7-7 Basic geometry of column*

- long and slender: \( L >> b, h \)
- constant cross-section (assumption is \( EI = \) constant)
b) **Loading** - Unlike a rod where the load is tensile, or compressive here the load is only compressive but it is still along the long direction (x_1 - axis)

c) **Deflection** - Here there is a considerable difference. Initially, it is the same as a rod in that deflection occurs along x_1 (u_1 -- shortening for compressive loads)

But we consider whether buckling (instability) can occur. In this case, we also have deflection transverse to the long axis, u_3. This u_3 is governed by bending relations:

\[
\frac{d^2 u_3}{dx_1^2} = \frac{M}{EI} \quad (u_3 = w)
\]
Figure M4.7-8 Representation of undeflected and deflected geometries of column

**undeflected:**

![Undeflected Free Body Diagram](image)

**deflected:**

![Deflected Free Body Diagram](image)

We again take a “cut” in the structure and use stress resultants:
Figure M4.7-9 Representation of “cut” column with resultant loads

Now use equilibrium:

\[ \sum F_1 = 0 \quad \Rightarrow \quad P + F(x_1) = 0 \]
\[ \Rightarrow \quad F(x_1) = -P \]

\[ \sum F_3 = 0 \quad \Rightarrow \quad S(x_1) = 0 \]

\[ \sum M_A = 0 \quad \Rightarrow \quad M(x_1) - F(x_1) u_3(x_1) = 0 \]
\[ \Rightarrow \quad M(x_1) + Pu_3(x_1) = 0 \]
Use the relationship between M and \( u_3 \) to get:

\[
EI \frac{d^2 u_3}{dx_1^2} + Pu_3 = 0
\]

This is the governing differential equation for Euler buckling (2nd order differential equation).

- always stabilizing (restoring)--basic beam:
- basic bending stiffness of structure resists deflection (pushes back)

- destabilizing for compressive load \( (u_3 > 0 \Rightarrow \text{larger force to deflect}) \);
- stabilizing for tensile load \( (F = -P) \) \( u_3 > 0 \Rightarrow \text{restoring force to get } u_3 = 0 \)

Note: + P is compressive

We now need to solve this equation and thus we look at the.....
(Solution for) Euler Buckling

First the

--> Basic Solution

(Note: may have seen similar governing for differential equation for harmonic notation:

\[
\frac{d^2 w}{dx^2} + k w = 0
\]

From Differential Equations (18.03), can recognize this as an eigenvalue problem. Thus use:

\[u_3 = e^{\lambda x_1}\]

Write the governing equation as:

\[
\frac{d^2 u_3}{dx_1^2} + \frac{P}{EI} u_3 = 0
\]
Note: will often see form
(differentiate twice for general B.C.’s)

\[
\frac{d^2}{dx_1^2} \left( EI \frac{d^2 u_3}{dx_1^2} \right) + \frac{d^2}{dx_1^2} (Pu_3) = 0
\]

This is more general but reduces to our current form if EI and P do not vary in \(x_1\)

Returning to:

\[
\frac{d^2 u_3}{dx_1^2} + \frac{P}{EI} u_3 = 0
\]

We end up with:

\[
\lambda^2 e^{\lambda x_1} + \frac{P}{EI} e^{\lambda x_1} = 0
\]

\[
\Rightarrow \lambda^2 = -\frac{P}{EI}
\]

\[
\Rightarrow \lambda = \pm \sqrt{\frac{P}{EI}} \quad i \quad \text{(also 0, 0 for 4th order Ordinary Differential Equation [O.D.E.]})
\]

where: \(i = \sqrt{-1}\)
We end up with the following general homogeneous solution:

\[ u_3 = A \sin \sqrt{\frac{P}{EI}} x_1 + B \cos \sqrt{\frac{P}{EI}} x_1 + C + D x_1 \]

comes from 4th order O.D.E. considerations

We get the constants A, B, C, D by using the **Boundary Conditions**

(4 constants from the 4th under O.D.E.  
⇒ need 2 B.C.’s at each end)

For the simply-supported case we are considering:

\[
\begin{align*}
@ x_1 = 0 & \left\{ \begin{array}{l}
    u_3 = 0 \\
    M = EI \frac{d^2u_3}{dx_1^2} = 0 \quad \Rightarrow \quad \frac{d^2u_3}{dx_1^2} = 0 \\
\end{array} \right. \\
\end{align*}
\]

\[
\begin{align*}
@ x_1 = L & \left\{ \begin{array}{l}
    u_3 = 0 \\
    M = EI \frac{d^2u_3}{dx_1^2} = 0 \quad \Rightarrow \quad \frac{d^2u_3}{dx_1^2} = 0 \\
\end{array} \right. \\
\end{align*}
\]
Note: \[ \frac{d^2 u_3}{dx_1^2} = - \frac{P}{EI} A \sin \sqrt{\frac{P}{EI}} x_1 - \frac{P}{EI} B \cos \sqrt{\frac{P}{EI}} x_1 \]

So using the B.C.’s:

\[ u_3 \left( x_1 = 0 \right) = 0 \Rightarrow B + C = 0 \]
\[ \frac{d^2 u_3}{dx_1^2} \left( x_1 = 0 \right) = 0 \Rightarrow B = 0 \]
\[ C = 0 \]

\[ u_3 \left( x_1 = L \right) = 0 \Rightarrow A \sin \sqrt{\frac{P}{EI}} L + DL = 0 \]
\[ \frac{d^2 u_3}{dx_1^2} \left( x_1 = L \right) = 0 \Rightarrow -A \sin \sqrt{\frac{P}{EI}} L = 0 \]

\[ \Rightarrow D = 0 \]

So we are left with:

\[ A \sin \sqrt{\frac{P}{EI}} L = 0 \]
This occurs if:

- \( A = 0 \) (trivial solution, \( \Rightarrow u_3 = 0 \))

- \( \sin \sqrt{\frac{P}{EI}} L = 0 \)

\[ \Rightarrow \sqrt{\frac{P}{EI}} L = n\pi \]  

integer

Thus, buckling occurs in a simply-supported column if:

\[ P = \frac{n^2 \pi^2 EI}{L^2} \]

associated with each load (eigenvalue) is a shape (eigenmode)

\[ u_3 = A \sin \frac{n\pi x}{L} \]

\textit{eigenvalues}

\textit{eigenmodes}
Note: A is still undefined. This is an instability \((u_3 \to \infty)\), so any value satisfies the equations.

[Recall, bifurcation is a mathematical concept]

Consider the buckling loads and associated mode shape (n possible)

*Figure M4.7-10* Potential buckling loads and modes for one-dimensional column

\[
P_3 = 9\pi^2EI/L^2
\]
\[
P_2 = 4\pi^2EI/L^2
\]
\[
P_1 = \pi^2EI/L^2
\]
The lowest value is the one where buckling occurs:

\[ P_{cr} = \frac{\pi^2 EI}{L^2} \]

**Euler (critical) buckling load (~1750)**

for simply-supported column

(Note: The higher critical loads can be reached if the column is “artificially restrained” at lower bifurcation loads)

There are also other configurations, we need to consider….

--> Other Boundary Conditions

There are 3 (/4) allowable Boundary Conditions on \( u_3 \) (need two on each end) which are **homogeneous** (B.C.’s…. = 0)
--- simply-supported

\( (\text{pinned}) \)

\[ u_3 = 0, \quad M = EI \frac{d^2u_3}{dx_1^2} = 0 \implies \frac{d^2u_3}{dx_1^2} = 0 \]

--- fixed end

\( (\text{clamped}) \)

\[ u_3 = 0, \quad \frac{du_3}{dx_1} = 0 \]

--- free end

\[ M = EI \frac{d^2u_3}{dx_1^2} = 0 \implies \frac{d^2u_3}{dx_1^2} = 0 \]

\[ S = 0 = \frac{dM}{dx_1} = \frac{d}{dx_1}\left(EI \frac{d^2u_3}{dx_1^2}\right) \implies \frac{d^3u_3}{dx_1^3} = 0 \]
--> sliding

\[
\begin{align*}
\frac{du_3}{dx_1} &= 0 \\
S &= 0 \quad \Rightarrow \quad \frac{d^3u_3}{dx_1^3} = 0
\end{align*}
\]

There are combinations of these which are inhomogeneous Boundary Conditions.

Examples…

--> free end with an axial load

\[
\begin{align*}
M &= 0 \\
S &= -P_0 \frac{du_3}{dx_1}
\end{align*}
\]
\( \Rightarrow \text{springs} \)

\( \begin{align*}
\text{(vertical)} & \quad S = k_f u_3 \\
\text{(torsional)} & \quad M = -k_f \frac{du_3}{dx_1}
\end{align*} \)

Need a general solution procedure to find \( P_{cr} \)

Do the same as in the basic case.

- same assumed solution \( u_3 = e^{\lambda x_1} \)
- yields basic general homogeneous solution

\[ u_3 = A \sin \sqrt{\frac{P}{EI}} x_1 + B \cos \sqrt{\frac{P}{EI}} x_1 + C + Dx_1 \]

- use B.C.’s (two at each end) to get four equations in four unknowns (A, B, C, D)
- solve this set of equations to find non-trivial value(s) of \( P \)
• set determinant of matrix to zero ($\Delta = 0$) and find roots (solve resulting equation)

roots = **eigenvalues** = buckling loads
also get associated……
**eigenmodes** = buckling shapes

--> will find that for homogeneous case, the critical buckling load has the generic form:

$$P_{cr} = \frac{c \pi^2 EI}{L^2}$$

where: $c =$ coefficient of edge fixity
→ depends on B.C.’s
For aircraft and structures, often use $c \approx 2$ for “fixed ends”.

Why?

- simply-supported is too conservative

- cannot truly get clamped ends

- actual supports are basically “torsional springs”, empirically $c = 2$ works well and remains conservative

$1 < c < 4$ (depends on $k_T$)
We’ve considered the “perfect” case of bifurcation where we get the instability in our mathematical model. Recall the opening example where that wasn’t quite the case. Let’s look at some realities here. First consider….

Effects of Initial Imperfections

We can think about two types…

Type 1 -- initial deflection in the column (due to manufacturing, etc.)

Figure M4.7-11 Representation of initial imperfection in column
**Type 2** -- load **not** applied along centerline of column

Define: \( e = \text{eccentricity} \) (downwards)

*Figure M4.7-12*  Representation of load applied off-line (eccentrically)

(a beam-column)

moment \( P_e \) plus axial load \( P \)

The two cases are basically handled the same way, but let’s consider Type 2 to illustrate…
The governing equation is still the same:
\[ \frac{d^2 u_3}{dx_1^2} + \frac{P}{EI} u_3 = 0 \]

Take a cut and equilibrium gives the same equations except there is an additional moment due to the eccentricity at the support: \( M = -Pe \)

Use the same basic solution:
\[ u_3 = A \sin \sqrt{\frac{P}{EI}} x_1 + B \cos \sqrt{\frac{P}{EI}} x_1 + C + Dx_1 \]

and take care of this moment in the Boundary Conditions:

Here:
\[ @ x_1 = 0 \left\{ \begin{array}{l} u_3 = 0 \quad \Rightarrow \quad B + C = 0 \\ M = EI \frac{d^2 u_3}{dx_1^2} = -Pe \quad \Rightarrow \quad -PB = -Pe \end{array} \right. \]
\[ B = e \]
\[ \Rightarrow C = -e \]

\[ @ \quad x_1 = L \begin{cases} u_3 = 0 \quad \Rightarrow \quad \ldots \end{cases} \]
\[ M = EI \frac{d^2 u_3}{dx_1^2} = -Pe \quad \Rightarrow \quad \ldots \]

Doing the algebra find:

\[ D = 0 \]

\[
A = \frac{e \left(1 - \cos \sqrt{\frac{P}{EI}} L\right)}{\sin \sqrt{\frac{P}{EI}} L} \quad \text{actual value for } A!
\]
Putting this all together:

\[ u_3 = e \left\{ \frac{1 - \cos \sqrt{\frac{P}{EI}L}}{\sin \sqrt{\frac{P}{EI}L}} \sin \sqrt{\frac{P}{EI}L} x_1 + \cos \sqrt{\frac{P}{EI}L} x_1 - 1 \right\} \]

**Notes:**
- Now get finite values of \( u_3 \) for values of \( P \).
- As \( P \to P_{cr} = \frac{\pi^2 EI}{L^2} \), still find \( u_3 \) becomes unbounded \((u_3 \to \infty)\).
Figure M4.7-13  Response of column to eccentric load

Nondimensionalize by dividing through by L

- Bifurcation is asymptote
- $u_3$ approaches bifurcation as $P \rightarrow P_{cr}$
- As $e/L$ (imperfection) increases, behavior is less like perfect case (bifurcation)

The other “deviation” from the model deals with looking at the general….
Failure of Columns

Clearly, in the “perfect” case, a column will fail if it buckles

\[ u_3 \rightarrow \infty \quad \text{(not very useful)} \]

\[ u_3 \rightarrow \infty \Rightarrow M \rightarrow \infty \Rightarrow \sigma \rightarrow \infty \Rightarrow \text{material fails!} \]

Let’s consider what else could happen depending on geometry

\[ \Rightarrow \text{ For long, slender case} \]

\[ P_{cr} = \frac{c\pi^2 EI}{L^2} \]

with:

\[ \sigma_{11} = \frac{P}{A} \]

\[ \Rightarrow \sigma_{cr} = \frac{c\pi^2 EI}{L^2 A} \text{ for buckling failure} \]
--> For short columns
if no buckling occurs, column fails when stress reaches material ultimate

\[ \sigma = \frac{P}{A} = \sigma_{cu} \]

\( \sigma_{cu} = \text{ultimate compressive stress} \)

failure by “squashing”

--> Behavior of columns of various geometries characterized via:

effective length: \( L' = \frac{L}{\sqrt{c}} \) (depends on Boundary Conditions)

radius of gyration: \( \rho = \sqrt{\frac{I}{A}} \) (ratio of moment of inertia to area)
Look at equation for $\sigma_{cr}$, can write as:

$$\sigma_{cr} = \frac{\pi^2 E}{(L'/\rho)^2}$$

Can capture behavior of columns of various geometries on one plot using: $\left(\frac{L'}{\rho}\right) = \text{“slenderness ratio”}$

*Figure M4.7-14 Representation of general behavior for columns of various slenderness ratios*

\[\begin{align*}
\sigma &\uparrow \\
\sigma_{cu} &\downarrow \\
\sigma_{cy} &\downarrow \\
\text{squashing} &\uparrow \\
\text{transition} &\uparrow \\
\text{buckling} &\downarrow \\
(\text{L'}/\rho) &\downarrow \\
\text{less slender} &\uparrow \\
\text{more slender} &\downarrow \\
\text{Euler curve:} &\quad \sigma_{cr} = \frac{\pi^2 E}{(L'/\rho)^2} \\
\text{where:} &\quad \sigma_{cy} = \text{compressive yield stress}
\end{align*}\]
Notes:

- for \( \left( \frac{L'}{\rho} \right) \) “large”, column fails by buckling
- for \( \left( \frac{L'}{\rho} \right) \) “small”, column squashes
- in transition region, plastic deformation (yielding) is taking place
  \[ \sigma_{cy} < \sigma < \sigma_{cu} \]

Let’s look at all this via an…

Example: a wood pointer-- assume it is pinned and about 4 feet long

**Figure M4.7-15** Geometry of pinned wood pointer
**Material properties:**
(Basswood)

\[ E = 1.4 \times 10^6 \text{ psi} \]
\[ \sigma_{cu} \approx 4800 \text{ psi} \]

--> Find maximum load P

**Step 1:** Find pertinent cross-section properties:

\[ A = b \times h = (0.25 \text{ in}) \times (0.25 \text{ in}) = 0.0625 \text{ in}^2 \]
\[ I = bh^3/12 = (0.25 \text{ in})(0.25 \text{ in})^3/12 = 3.25 \times 10^{-4} \text{ in}^4 \]

**Step 2:** Check for buckling

use:

\[ P_{cr} = \frac{c\pi^2 EI}{L^2} \]

simply-supported \( \Rightarrow \) \( c = 1 \)
So: \[ P_{cr} = \frac{\pi^2(1.4 \times 10^6 \text{ lbs/in}^2)(3.26 \times 10^{-4} \text{ in}^{-4})}{(48 \text{ in})^2} \]

\[ \Rightarrow P_{cr} = 1.96 \text{ lbs} \]

**Step 3:** Check to see if it buckles or squashes

\[ \sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.96 \text{ lbs}}{0.0625 \text{ in}^2} = 31.4 \text{ psi} \]

So: \( \sigma_{cr} < \sigma_{cu} \Rightarrow \text{BUCKLING!} \)

--> **Variations**

1. What is “transition” length?

Determine where “squashing” becomes a concern (approximately)

\[ \Rightarrow \sigma_{cr} = \sigma_{cu} \]
\[ \sigma_{cr} = \frac{P_{cr}}{A} = 4800 \ \text{psi} \]
\[ \Rightarrow \quad P_{cr} = \left(4800 \ \text{lbs} / \text{in}^2\right)(0.0625 \ \text{in}^2) \]
\[ \Rightarrow \quad P_{cr} = 300 \ \text{lbs} \]

\[ \Rightarrow \text{work backwards} \]

\[ P_{cr} = \frac{\pi^2 EI}{L^2} \]

where \( L \) is the variable, gives:

\[ L^2 = \frac{\pi^2 EI}{P_{cr}} \]
\[ \Rightarrow \quad L = \sqrt{\frac{\pi^2 \left(1.4 \times 10^6 \ \text{lbs} / \text{in}^2\right)(3.26 \times 10^{-4} \ \text{in}^4)}{300 \ \text{lbs}}} \]
\[ \Rightarrow \quad L = \sqrt{15.01 \ \text{in}^2} \quad \Rightarrow \quad L = 3.87 \ \text{in} \]
Finally....

If $L > 3.87$ in $\Rightarrow$ buckling
If $L < 3.87$ in $\Rightarrow$ squashing

Note transition “around” 3.87 in due to yielding (basswood relatively brittle)

**Figure M4.7-16** Behavior of basswood pointer subjected to compressive load
2. What if rectangular cross-section?

![Diagram of rectangular cross-section with dimensions 0.5" x 0.25".]

Does it still buckle in $x_3$ - direction?

Consider I about $x_2$ - axis and $x_3$ - axis

--> $x_2$ - axis ⇒ $h = 0.5$ in, $b = 0.25$ in
\[
\Rightarrow I_2 = \frac{bh^3}{12} = \frac{(0.25 \text{ in})(0.50 \text{ in})^3}{12} = 0.0026 \text{ in}^4 = 2.60 \times 10^{-3} \text{ in}^4
\]

\[

gp x_3 - \text{axis} \Rightarrow
\]

\[
\Rightarrow h = 0.25 \text{ in}, \quad b = 0.5 \text{ in}
\]

\[
I_3 = \frac{bh^3}{12} = \frac{(0.50 \text{ in})(0.25 \text{ in})^3}{12} = 0.00065 \text{ in}^4 = 0.65 \times 10^{-3} \text{ in}^4
\]

then use:

\[
P_{cr} = \frac{\pi^2 EI}{L^2}
\]
and find:

\[ I_3 < I_2 \]

\[ \Rightarrow P_{cr} \text{ smaller for buckling about } x_3 \text{- axis.} \]

For buckling, h is the shorter/smaller cross-section dimension since buckling occurs about axis with smallest \( I \)!

--- Final note on buckling

...possibility of occurrence in any structure where there is a compressive load (thinner structures most susceptible)
Unit M4.7 (New) Nomenclature

c -- coefficient of edge fixity
e -- eccentricity (due to loading off line or initial imperfection)
$I_2$ -- moment of inertia about $x_2$ - axis
$I_3$ -- moment of inertia about $x_3$ - axis
$k_{\text{eff}}$ -- effective stiffness
$k_f$ -- axial stiffness
$k_T$ -- torsional stiffness
$L$ -- effective length (in buckling considerations)
$L'/\rho$ -- slenderness ratio
$P$ -- compressive load along column
$P_{\text{cr}}$ -- critical (buckling) load (for instability)
$\rho$ -- radius of gyration (square root of ratios of moment of inertia to area)
$\sigma_{\text{cr}}$ -- critical buckling stress
$\sigma_{\text{cu}}$ -- compressive ultimate stress
$\sigma_{\text{cy}}$ -- compressive yield stress