PRODUCTION OF THRUST

Newton’s 2nd Law (e.g. \( \dot{F} = d/dt (mv) \))
for a control volume of fixed mass with steady flow in and out
and no acceleration of the frame of reference relative to inertial
coordinates:

\[
\bar{F} = \int_{s} \bar{u}(\bar{u} \cdot \bar{n}) ds
\]

Sum of external forces
on control volume, e.g.
Pressure forces
Shear forces
Body forces
Reaction forces

Net flux of momentum through
surface of control volume

For x-component of vectors:

\[
F_x = \int_{s} u_x \bar{u} \cdot \bar{n} ds
\]
PRODUCTION OF THRUST

\[ F \text{ (reaction)} = T \cdot D \]

(A\text{\_R} = \text{rest of area not taken by } A\text{\_e})

\[ T \cdot D + \int \text{Pressure Forces} = \int_{s} \overrightarrow{u}(\overrightarrow{u} \cdot \overrightarrow{n})dA \]

\[ T \cdot D + \int P\text{\_e}A\text{\_e} + P\text{\_o}A\text{\_e} \int_{A\text{\_R}} (P\text{\_R} - P\text{\_o}) dA = \int_{s} \overrightarrow{u}(\overrightarrow{u} \cdot \overrightarrow{n})dA \]

\[ \int_{s} \overrightarrow{u} \cdot \overrightarrow{n}dA = \int_{s} \overrightarrow{u} \cdot \overrightarrow{n}dA \]

\[ \int_{s} \overrightarrow{u} \cdot \overrightarrow{n}dA = \int_{s} \overrightarrow{u} \cdot \overrightarrow{n}dA \]

\[ T \cdot D \int (P\text{\_e} - P\text{\_o}) A\text{\_e} \int_{A\text{\_R}} (P\text{\_R} - P\text{\_o}) dA = \int_{s} \overrightarrow{u}(\overrightarrow{u} \cdot \overrightarrow{n})dA \]

\[ \dot{m}\text{\_e} \overrightarrow{u} \cdot \overrightarrow{n}dA + \int_{c\text{\_s} \cdot A\text{\_o} \cdot A\text{\_e}} \overrightarrow{u}(\overrightarrow{u} \cdot \overrightarrow{n})dA \]
Everything that relates to flow through the engine is conventionally called **thrust**. Everything that relates to the flow on the outside of the engine is conventionally call **drag**. Therefore, gathering only those terms that relate to the fluid that passes through the engine, we have:

\[
T = \dot{m}_e u_e - \dot{m}_o u_o + (P_e - P_o)A_e
\]

The **thrust** is largely composed of the net change in momentum of the air entering and leaving the engine, with a typically small adjustment for the differences in pressure between the inlet and the exit.
EFFICIENCY

We have related the thrust of a propulsion system to the net changes in momentum, pressure forces, etc. Now we will look at how efficiently the propulsion system converts one form of energy to another on its way to producing thrust:

\[
\text{overall efficiency} = \frac{\text{what you get}}{\text{what you pay for}} = \frac{\text{propulsive power}}{\text{fuel power}}
\]

\[
\text{propulsive power} = \text{thrust} \cdot \text{flight velocity} = T_u \circ
\]

\[
\text{fuel power} = \text{fuel mass flow rate} \cdot \text{fuel energy per unit mass} = \dot{m}_f h
\]

Thus,

\[
\eta_{\text{overall}} = \frac{T_u \circ}{\dot{m}_f h}
\]

Waitz, 2002
EFFICIENCY

It is often convenient to break the overall efficiency into:
thermal efficiency and propulsive efficiency where

\[
\eta_{\text{thermal}} = \frac{\text{rate of production of propellant k.e.}}{\text{fuel power}} = \frac{\dot{m}_e u_e^2 - \dot{m}_o u_o^2}{\dot{m}_f h}
\]

\[
\eta_{\text{prop}} = \frac{\text{propulsive power}}{\text{rate of production of propellant k.e.}} = \frac{Tu_o}{\dot{m}_e u_e^2 - \dot{m}_o u_o^2}
\]

Such that,

\[
\eta_{\text{overall}} = \eta_{\text{thermal}} \eta_{\text{propulsive}}
\]
Trends in thermal efficiency are driven by increasing compression ratios and corresponding increases in turbine inlet temperature. Whereas trends in propulsive efficiency are due to generally higher bypass ratio engines.

(After Koff, 1991)
The **thermal efficiency** is the same as that used in thermodynamics. For an ideal Brayton cycle it is a function of the temperature ratio across the compressor

\[
\text{\%th\text{idealBraytoncycle}} = \frac{W_{\text{net}}}{Q_{\text{in}}} = 1 - \frac{T_{\text{atm}}}{T_{\text{comp exit}}} = 1 - \frac{1}{(\text{PR})^{\frac{1}{g - 1}}}
\]

Higher temperature ratio = higher pressure ratio = higher thermal efficiency
TURBINE INLET TEMPERATURE

- Desire for higher turbine inlet temperature is driven by desire for high specific work

- Specific work is work per unit of airflow

- High work per unit of airflow = smaller engine, lower weight, etc.
For a given turbine inlet temperature, there is a compressor pressure ratio that maximizes the work per unit of airflow.

\[ TR = \frac{T_{T4}}{T_0} \]
TURBINE INLET TEMPERATURE TRENDS

Hydrocarbon Stoichiometric Limit

Ideal Brayton Cycle Performance

Von Ohain (1939)
Whittle (1937)
JT8D J52 PW2037 F100
J47 J58 PW4000 TF30
J57 J42

Specific Core Power (hp/lb/sec)

Turbine Rotor Inlet Temperature (°F)

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PROPULSIVE EFFICIENCY

We can use our expression for thrust to rewrite the equation for propulsive efficiency in a more convenient form

\[ T = \dot{m}(u_e \gamma u_o) \quad (\text{since } \dot{m}_e = \dot{m}_o) \]

Then,

\[ \Pi_p = \frac{\dot{m}u_o(u_e \gamma u_o)}{\frac{\dot{m}}{2}(u_e^2 \gamma u_o^2)} = \frac{2u_o}{u_o + u_e} = \frac{2}{1 + \frac{u_e}{u_o}} \]
TRENDS IN BYPASS RATIO

(After Koff, 1991)
OTHER EXPRESSIONS FOR OVERALL EFFICIENCY

Specific Impulse ($I$ or $I_{sp}$):

$$I = \frac{\text{thrust}}{\text{fuel weight flow rate}}$$

(units: seconds)

Thrust Specific Fuel Consumption (SFC or TSFC):

$$\text{SFC} = \frac{\text{mass flow rate of fuel}}{\text{thrust}}$$

(units: lbm/hr/lbf or kg/s/N)
IMPLICATIONS FOR ENGINE DESIGN

Considering jointly the expressions for thrust and propulsive efficiency,

\[ F = \dot{m}(u_e - u_o) \]

\[ \eta_{prop} = \frac{2}{1 + \frac{u_e}{u_o}} \]

As \[ \frac{u_e}{u_o} \uparrow \]

\[ \frac{F}{\dot{m}} \uparrow \eta_{prop} \]

As \[ \frac{u_e}{u_o} \downarrow 1 \]

\[ \frac{F}{\dot{m}} \downarrow \eta_{prop} \uparrow \]

Also, as \[ \frac{F}{\dot{m}u_o} \uparrow \]

\[ A_{inlet} \]

Drag

\[ u_o \rightarrow A_{inlet} \]

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PROPULSIVE EFFICIENCY AND SPECIFIC THRUST

\[ \eta_p \]

\[ \frac{u_e}{u_0} \]

\[ \frac{F}{\dot{m} u_0} \]

PROPULSIVE EFFICIENCY VS. FLIGHT SPEED

![Graph showing propulsive efficiency vs. flight speed. The graph compares different types of jet engines, including Turbo-Prop, Low-By-Pass Turbo-Jet, and High-By-Pass Turbo-Jet. The efficiency is shown in percentage, and the speed is measured in miles per hour (m.p.h.).]
PROPULSIVE EFFICIENCY AND SPECIFIC THRUST

For fighter aircraft that need high thrust/weight and fly at high speed, it is typical to employ engines with smaller inlet areas and higher thrust per unit mass flow.

However, transport aircraft that require higher efficiency and fly at lower speeds usually employ engines with relatively larger inlet areas and lower thrust per unit mass flow.
COMMERCIAL AND MILITARY ENGINES
(Approx. same thrust, approx. correct relative sizes)

GE CFM56 for Boeing 737

PW JSF119 for Joint Strike Fighter
PROPULSIVE EFFICIENCY

At low flight velocities, the highest propulsive efficiency is typically obtained with a propeller or an unducted fan.

(d) Prop-fan concept

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AIRCRAFT AND ENGINE DESIGN ISSUES

• Thrust \( \approx \) (mass flow) \( \times \) (change in velocity across engine)

• Range \( \sim \) fuel efficiency (commercial and military)
  – Thermal efficiency \( \{ \) High pressures and temperatures \( \}
  – Propulsive efficiency \( \{ \) Large mass flow with small velocity change \( \}

• Maneuverability (military)
  – High thrust-per-weight, small compact engine \( \{ \) High energy conversion per unit volume (high temperatures and pressures) \( \}

• Supersonic flight (military)
  – Low drag, small compact engine \( \{ \) Small mass flow with large velocity change \( \)
AN EXAMPLE OF CYCLE PERFORMANCE IMPROVEMENT

\[ \Delta \text{Core Thermal Efficiency} \]
\[ \eta_T = \frac{\text{power}}{\text{heat added}} \]

\[ \Delta \text{Propulsive Efficiency} \]
\[ \eta_p = \frac{\text{thrust}}{\text{core power}} \times \text{aircraft velocity} \]