Due before 4PM on Wednesday November 26, 2003, in the boxes in 2-106. No late homework will be accepted. There is one box for each recitation section. For full credit, please be sure to show and explain your work.

1. Suppose $A$ is a 3 by 3 symmetric matrix with unit eigenvectors $u_1$, $u_2$, and $u_3$. If its eigenvalues are $\lambda_1 = 2$, $\lambda_2 = 1$, $\lambda_3 = -1$, what are the matrices $U$, $\Sigma$, and $V^T$ in its SVD?

2. Let $J = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and $K = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

   (a) What are the eigenvalues of $J$ and $K$?
   (b) Show that $J$ is not similar to $K$.

3. For each of the following statements, state whether the statement is true or false. If the statement is true, explain why it is true. If the statement is false, give a counterexample to the statement (i.e. give an specific example for which the statement is incorrect and show that the statement is false for that example).

   (a) If $A$ is similar to $B$, then $A^2$ is similar to $B^2$.
   (b) If $A^2$ is similar to $B^2$, then $A$ is similar to $B$.
   (c) $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ is similar to $\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$
   (d) If we exchange rows 1 and 2 of $A$, and then exchange columns 1 and 2, the eigenvalues stay the same.

4. Suppose a linear transformation $T$ transforms $(1,1)$ to $(2,2)$ and $(2,1)$ to $(0,0)$. Find $T(v)$ when

   (a) $v = (5,3)$
   (b) $v = (0,1)$.

5. Let $V$ denote the vector space of all 2 by 2 matrices. Let $T$ denote the function defined by

$$T(M) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} M \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

for all $M$ in $V$.

   (a) Show that $T$ is a linear transformation.
   (b) What is the dimension of the range of $T$?
   (c) Describe all matrices in the kernel of $T$. 