Your PRINTED name is: ____________________

Please circle your recitation:

1) M 2 2-131 P. Lee 2-087 2-1193 lee
2) M 2 2-132 T. Lawson 4-182 8-6895 tlawson
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9) T 1 2-132 A. Chan 2-588 3-4110 alicec
10) T 1 2-131 D. Chebikin 2-333 3-7826 chebikin
11) T 2 2-132 A. Chan 2-588 3-4110 alicec
12) T 3 2-132 T. Lawson 4-182 8-6895 tlawson
1 (30 pts.) The matrix $A$ has a varying $1 - x$ in the $(1, 2)$ position:

$$A = \begin{bmatrix}
2 & 1 - x & 0 & 0 \\
1 & 1 & 1 & 1 \\
1 & 1 & 2 & 4 \\
1 & 1 & 3 & 9
\end{bmatrix}$$

(a) When $x = 1$ compute $\det A$. What is the $(1, 1)$ entry in the inverse when $x = 1$?

(b) When $x = 0$ compute $\det A$.

(c) How do the properties of the determinant say that $\det A$ is a linear function of $x$? For any $x$ compute $\det A$. For which $x$'s is the matrix singular?
2 (30 pts.) This matrix $Q$ has orthonormal columns $q_1, q_2, q_3$:

$$
Q = \begin{bmatrix}
.1 & .5 & a \\
.7 & .5 & b \\
.1 & -.5 & c \\
.7 & -.5 & d \\
\end{bmatrix}
$$

(a) What equations must be satisfied by the numbers $a, b, c, d$? Is there a unique choice for those numbers, apart from multiplying them all by $-1$?

(b) Why is $P = QQ^T$ a projection matrix? (Check the two properties of projections.) Why is $QQ^T$ a singular matrix? Find the determinants of $Q^TQ$ and $QQ^T$.

(c) Suppose Gram-Schmidt starts with those same first two columns and with the third column $a_3 = (1, 1, 1)$. What third column would it choose for $q_3$? You may leave a square root not completed (if you want to).
3 (40 pts.) Our measurements at times $t = 1, 2, 3$ are $b = 1, 4,$ and $b_3$. We want to fit those points by the nearest line $C + Dt$, using least squares.

(a) Which value for $b_3$ will put the three measurements on a straight line? 
Whi ch line is it? Will least squares choose that line if the third measurement is $b_3 = 9$? (Yes or no).

(b) What is the linear system $Ax = b$ that would be solved exactly for $x = (C, D)$ if the three points do lie on a line? Compute the projection matrix $P$ onto the column space of $A$. Remember the inverse

$$
\begin{bmatrix}
a & b \\
c & d
\end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.
$$

(c) What is the rank of that projection matrix $P$? How is the column space of $P$ related to the column space of $A$? (You can answer with or without the entries of $P$ computed in (b).)

(d) Suppose $b_3 = 1$. Write down the equation for the best least squares solution $\tilde{x}$, and show that the best straight line is horizontal.
xxx