Problem 1 Monday 9/18
Look at Worked Example 3.1B from section 3.1 in your book.
For each of the vector spaces \( V_1 \) through \( V_4 \),
describe a subspace, different from the examples in the text, in two different ways:
all combinations of \ldots = all solutions to \ldots

Problem 2 Monday 9/18
(a) Do Problem #19 from section 3.1 in your book.
(b) Also describe the nullspaces of each matrix.

Problem 3 Wednesday 9/20
Suppose the \( m \)-by-\( n \) matrix \( A \) (\( m < n \)) has a right inverse \( B \), that is, a matrix \( B \) such that \( AB = I \),
the identity.
(a) What must the dimensions of \( B \) and of \( I \) be?
(b) Try calculating \( B \) in Matlab: let \( A = \begin{bmatrix} 2 & 3 & -5 \\ 0 & -1 & 2 \end{bmatrix} \) and find \( A \backslash I \). (The identity is \texttt{eye(k)} \) in Matlab.)
(c) Now try calculating \( B \) another way, with \texttt{rref([A I])}. (This is the reduced-row echelon form,
the result of Gauss-Jordan elimination.) What do you get? Now state another, different, \( B \) with \( AB = I \). (Hint: Not all the rows of \( B \) are shown, unlike the square case.)
(d) Why can’t there be a left inverse \( CA = I \)? And what would the dimensions of \( C \) and \( I \) be if there were?

Problem 4 Wednesday 9/20
Do Problem #23 from section 3.2 in your book.

Problem 5 Wednesday 9/20
Let \( v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \). Describe the nullspace of the matrix \( vv^\top \) geometrically.

Problem 6 Friday 9/22
Do Problem #3 from section 3.3 in your book.
Problem 7 Friday 9/22

Let \[ A = \begin{bmatrix} -1 & 2 & 5 & 0 & 5 \\ 2 & 1 & 0 & 0 & -15 \\ 6 & -1 & -8 & -1 & -47 \\ 0 & 2 & 4 & 3 & 16 \end{bmatrix}. \]

(a) Reduce \( A \) to (ordinary) echelon form.
(b) What are the pivots? What are the free variables?
(c) Now reduce \( A \) to row-reduced echelon form.
(d) Give the special solutions. What is the nullspace \( N(A) \)?
(e) What is the rank of \( A \)?
(f) Give the complete solution to \( Ax = b \), where \( b = A \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \).

Problem 8 Friday 9/22

Do Problem #5 from section 3.4 in your book.

Problem 9 Friday 9/22

Do Problem #13 from section 3.4 in your book.
(Answer in back of book, but try to do it yourself first.)

Problem 10 Friday 9/22

Do Problem #32 from section 3.4 in your book.

\(^1\)Modified 12/24—thanks to Laura Garrity for pointing out the original solution’s error. Hopefully it’s fixed now.