Your PRINTED name is:  

Please circle your recitation:  

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 1) | T 10 | 2-131 | K. Meszaros | 2-333 | 3-7826 | karola |
| 2) | T 10 | 2-132 | A. Barakat  | 2-172 | 3-4470 | barakat|
| 3) | T 11 | 2-132 | A. Barakat  | 2-172 | 3-4470 | barakat|
| 4) | T 11 | 2-131 | A. Osorno   | 2-229 | 3-1589 | aosorno|
| 5) | T 12 | 2-132 | A. Edelman  | 2-343 | 3-7770 | edelman|
| 6) | T 12 | 2-131 | K. Meszaros | 2-333 | 3-7826 | karola |
| 7) | T 1  | 2-132 | A. Edelman  | 2-343 | 3-7770 | edelman|
| 8) | T 2  | 2-132 | J. Burns    | 2-333 | 3-7826 | burns  |
| 9) | T 3  | 2-132 | A. Osorno   | 2-229 | 3-1589 | aosorno|
This question is about an $m$ by $n$ matrix $A$ for which

$$Ax = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ has no solutions} \quad \text{and} \quad Ax = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ has exactly one solution}.$$ 

(a) Give all possible information about $m$ and $n$ and the rank $r$ of $A$.

(b) Find all solutions to $Ax = 0$ and explain your answer.

(c) Write down an example of a matrix $A$ that fits the description in part (a).
The 3 by 3 matrix $A$ reduces to the identity matrix $I$ by the following three row operations (in order):

- $E_{21}$: Subtract 4 (row 1) from row 2.
- $E_{31}$: Subtract 3 (row 1) from row 3.
- $E_{23}$: Subtract row 3 from row 2.

(a) Write the inverse matrix $A^{-1}$ in terms of the $E$'s. Then compute $A^{-1}$.

(b) What is the original matrix $A$?

(c) What is the lower triangular factor $L$ in $A = LU$?
This page intentionally blank.
This 3 by 4 matrix depends on $c$:

$$A = \begin{bmatrix}
1 & 1 & 2 & 4 \\
3 & c & 2 & 8 \\
0 & 0 & 2 & 2
\end{bmatrix}$$

(a) For each $c$ find a basis for the column space of $A$.

(b) For each $c$ find a basis for the nullspace of $A$.

(c) For each $c$ find the complete solution $x$ to $Ax = \begin{bmatrix} 1 \\ c \\ 0 \end{bmatrix}$. 
This page intentionally blank.
4 (24 pts.) (a) If $A$ is a 3 by 5 matrix, what information do you have about the nullspace of $A$?

(b) Suppose row operations on $A$ lead to this matrix $R = \text{rref}(A)$:

$$R = \begin{bmatrix} 1 & 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Write all known information about the columns of $A$.

(c) In the vector space $M$ of all 3 by 3 matrices (you could call this a matrix space), what subspace $S$ is spanned by all possible row reduced echelon forms $R$?
This page intentionally blank.