1. a) Subtracting $1806 \times 1$st row from 2nd row and $2013 \times 1$st row from 3rd row, matrix becomes

\[
\begin{pmatrix}
1 & 1 & 1 \\
0 & 0 & -1806 \\
0 & 1 & 2
\end{pmatrix}
\]

Then det $= 1806$ clearly. (Or any correct method).

b) Subtract 1st row from each other row to get

\[
\begin{pmatrix}
1 & \cdots & \cdots & \cdots & 1 \\
0 & 1 & \cdots & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & \cdots & 0 & \cdots & 1
\end{pmatrix}
\]

So det $A_n = (-1)^{n-1}$. (Or any correct method).
2. (a) $P$ is projection onto a 3 dimensional subspace of an $m \geq 3$-dimensional vector space. So $\det(P) = 0$.

b) The eigenvalues are $1$ (multiplicity 3) and $0$ (multiplicity $m - 3$).

c) An eigenvector is $q_1$, with eigenvalue $q_1^T q_1 = q_1 \cdot q_1$.

d) $P$ is symmetric, so $M$ is symmetric.

Left nullspace = right nullspace = column space of $P$

Column space = row space = orthogonal complement of column space of $P$
3. a) Quartic, since the diagonal term in the big formula is \((a_{11} - x)(a_{22} - x)(a_{33} - x)(a_{44} - x)\).

b) Linear, since \(A_{11}\) appears at most once in any term of the big formula.

c) Quartic, since

\[
\det(xA) = x^4 \det(A).
\]

d) Quadratic: we can add the second row to the third row to eliminate some \(x\)s. Then each term of the big formula is quadratic in \(x\).
4. a) Let \( M = (A \ B) \) be the \( 3 \times 2 \) matrix with columns \( A \) and \( B \). Since \( A \) and \( B \) are orthonormal, the projection onto the \((A, B)\)-plane is given by \( MM^T \).

Thus the length \( L \) is the length of \( C - MM^T C \)

\[
L = \| C - MM^T C \|.
\]

b) The volume of a pyramid is

\[
\frac{1}{3} \text{ (base area)(altitude length)}
\]

The area \( \Theta \) of the OAB face is \( \frac{1}{2} \), so the volume is

\[
V = \frac{1}{6} L = \frac{1}{6} \| C - MM^T C \|.
\]