Your PRINTED name is: ____________________________

Please circle your recitation:

1  T 9   Dan Harris   E17-401G   3-7775   dmh
2  T 10  Dan Harris   E17-401G   3-7775   dmh
3  T 10  Tanya Khovanova   E18-420   4-1459   tanya
4  T 11  Tanya Khovanova   E18-420   4-1459   tanya
5  T 12  Saul Glasman   E18-301H   3-4091   sglasman
6  T 1   Alex Dubbs   32-G580   3-6770   dubbs
7  T 2   Alex Dubbs   32-G580   3-6770   dubbs
This page intentionally blank.
1 (32 pts.) (2 points each)

There are sixteen $2 \times 2$ matrices whose entries are either 0 or 1. For each of the sixteen, write down the two singular values. Time saving hint: if you really understand singular values, then there is really no need to compute $AA^T$ or $A^TA$, but it is okay if you must.
This page intentionally blank.
This page intentionally blank.
This page intentionally blank.
2 (30 pts.) (3 points each: Please circle true or false, and either way, explain briefly.)

a) If $A$ and $B$ are invertible, then so is $(A + B)/2$. True? False? (Explain briefly).

b) If $A$ and $B$ are Markov, then so is $(A + B)/2$. True? False? (Explain briefly).

c) If $A$ and $B$ are positive definite, then so is $(A + B)/2$. True? False? (Explain briefly).

d) If $A$ and $B$ are diagonalizable, then so is $(A + B)/2$. True? False? (Explain briefly).

e) If $A$ and $B$ are rank 1, then so is $(A + B)/2$. True? False? (Explain briefly).
f) If $A$ is symmetric then so is $e^A$.

True?  False? (Explain briefly).

g) If $A$ is Markov then so is $e^A$.

True?  False? (Explain briefly).

h) If $A$ is symmetric, then $e^A$ is positive definite.

True?  False? (Explain briefly).

i) If $A$ is singular, then so is $e^A$.

True?  False? (Explain briefly).

j) If $A$ is orthogonal, then so is $e^A$.

True?  False? (Explain briefly).
3 (38 pts.)

Let \( A = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \).

a) (10 pts.) Find a nonzero solution \( y(t) \) in \( \mathbb{R}^2 \) to \( \frac{dy}{dt} = Ay \) that is independent of \( t \), in other words, \( y(t) \) is a constant vector in \( \mathbb{R}^2 \). (Hint: why would a vector in the nullspace of \( A \) have this property?)

b) (10 pts.) Show that \( e^{At} \) is Markov for every value of \( t \geq 0 \).
c) (10 pts.) What is the limit of $e^{At}$ as $t \to \infty$?

d) (8 pts.) What is the steady state vector of the Markov matrix $e^A$?