a. Using the matrix squaring operator create a “triangular” matrix with 1 on the main diagonal, 2 above, etc.

\[
M(n) = \begin{pmatrix}
1 & 2 & \ldots & n-1 & n \\
1 & 2 & \ldots & n-1 & \cdot \\
& \ddots & \ddots & \ddots & \ddots \\
& & 1 & 2 & 1 \\
& & & 1 & 1
\end{pmatrix}
\]

In[1]: # The line below is a Julia function that given n computes M(n)
# If you are not using Julia please supply your function
M(n) = triu(ones(n,n))^2
# Show that it works
M(5)

Out[1]: # Put your output here

In[2]: # Show that it works for n=1,2,3,4,5,6
{M(n) for n=1:6}

Out[2]: # Put your output here
b. You very likely have heard of the triangular numbers: (see wikipedia if not)

\[ T_n = 1 + 2 + \ldots + n. \]

\[
M(n) = \begin{pmatrix}
1 & 3 & \ldots & (n-1)n/2 & n(n+1)/2 \\
1 & 3 & \ldots & (n-1)n/2 \\
& \ddots & \ddots \\
& & 1 & 3 \\
& & & 1
\end{pmatrix}
\]

Don't use cumsum, or sum or “+”, just matrix operations to create the matrix that has the triangular numbers on the diagonals: Explain roughly (not too formal a proof), why your idea works.

\[
In[3]: \text{# Here they are}
\text{cumsum(1:10)'}
# If not using Julia, how would you do this in your language?
\]

\[
Out[3]: 1x10 Array(Int64, 2):
[1 3 6 10 15 21 28 36 45 55]
\]

\[
In[4]: \text{# Put your input code here, put in a function and show that it}
\text{works, just like In[1] and Out[1]}
\]

\[
Out[4]:
\]
c. Don’t stop. Keep going, and get the tetrahedral numbers. (see wikipedia) Explain briefly why this worked.

\[
\text{In}[5]: \quad \# \text{ This is hardly much different from In[4] and Out[4]} \\
\text{Out}[5]: \\
\]
d. Let \( A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 1 \end{bmatrix} \) and \( B = \begin{bmatrix} -1 & 2 & 1 & 4 \end{bmatrix} \).

Compute \((AB)^{10}\) on your computer. Explain why it’s possible to get this without a computer!

In[6]: A=[1;2;3;1]

Out[6]: 4-element Array{Int64,1}:
1
2
3
1

In[7]: B=[-1 2 1 4]

Out[7]: 1x4 Array{Int64,2}:
-1 2 1 4

In[8]: (A*B)^10

Out[8]: # Put your solution here

In[9]: B*A

Out[9]: # Put your solution here

and see if you can see and tell us what is going on