18.06 Midterm Exam 2, Spring, 2001

Name ____________________________ Optional Code ______________________
Recitation Instructor __________________ Email Address ____________________
Recitation Time ____________________

This midterm is closed book and closed notes. No calculators, laptops, cell phones or other electronic devices may be used during the exam.
There are 3 problems. Good luck.

1. (40pts.) Consider the matrix

\[
A = \begin{pmatrix}
1 & 0 & -1 \\
3 & 1 & -1 \\
9 & 5 & 1 \\
9 & 8 & 7
\end{pmatrix}
\]

(a) Find the rank of \( A \).

(b) Find a basis for the row space of \( A \), and find a basis for the nullspace of \( A \). What is the dimension of the nullspace of \( A \)?

(c) What can you say about the relation between the rank and the dimension of the nullspace of \( A \)?

(d) Verify that all vectors in your basis of the nullspace are orthogonal to all vectors in your basis of the row space.

2. (30pts.) Let \( a, b \in \mathbb{R} \), and let

\[
A = \begin{pmatrix}
1 & 2 & 3 & a \\
1 & 0 & -1 & 0 \\
0 & 1 & 2 & b
\end{pmatrix}.
\]

(a) What are the dimensions of the four subspaces associated with the matrix \( A \)? This will of course depend on the values of \( a \) and \( b \), and you should distinguish all different cases.

(b) For \( a = b = 1 \), give a basis for the column space of \( A \). Is this also a basis for \( \mathbb{R}^3 \)? Justify your answer.

3. (30pts.) An experiment at the seven times \( t = -3, -2, -1, 0, 1, 2, 3 \) yields the consistent result \( b = 0 \), except at the last time \( (t = 3) \), when we get \( b = 28 \). We want the best straight line \( b = C + Dt \) to fit these seven data points by least squares.

(a) Write down the equation \( Ax = b \) with unknowns \( C \) and \( D \) that would be solved if a straight line exactly fit the data.
(b) Use the method of least squares to find the best fit values for \( C \) and \( D \).

(c) This problem is really that of projecting the vector \( \mathbf{b} = (0,0,0,0,0,28)^T \) onto a certain subspace. Give a basis for that subspace, and give the projection \( \mathbf{p} \) of \( \mathbf{b} \) onto that subspace.