18.06 Spring 2006 - Problem Set 6

SOLUTIONS TO SELECTED PROBLEMS

1. Section 5.1, Problem 8

Answer: There are $5! = 120$ permutation matrices. $5!/2 = 60$ have det = +1.

A permutation matrix that needs four exchanges to reach the identity matrix:

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}
$$

2. Section 5.2, Problem 25

Answer:

a) If we use the big formula to find the determinant, picking an entry from $B$ requires picking an entry from the zero block which results in zero. This leaves a pair of entries from $A$ times a pair from $D$ leading to $(\det A)(\det D)$.

b) and c)

$$
A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}
$$
3. Section 5.3, Problem 1

**Answer:**

a) $\det A = 3$, $\det B_1 = -6$, $\det B_2 = 3$. Therefore $x_1 = -6/3 = -2$ and $x_2 = 3/3 = 1$.

b) $\det A = 4$, $\det B_1 = 3$, $\det B_2 = -2$, $\det B_3 = 1$. Therefore $x_1 = 3/4$, $x_2 = -1/2$, $x_3 = 1/4$.

4. Section 5.3, Problem 7

**Answer:** If all the cofactors are 0, then $\det A = 0$ (by the Cofactor Formula for determinants) and $A$ has no inverse.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$ has no zero cofactors but it is not invertible.