1. Section 4.1, Problem 19

Answer: Suppose \( L \) is a one-dimensional subspace in \( \mathbb{R}^3 \). It’s orthogonal complement \( L^\perp \) is the **2-dimensional subspace (a plane)** perpendicular to \( L \). The \( (L^\perp)^\perp \) is a **1-dimensional subspace (a line)** perpendicular to \( L^\perp \). In fact \( (L^\perp)^\perp \) is the same as \( L \).

2. Section 4.1, Problem 20

Answer: Suppose \( V \) is the whole space \( \mathbb{R}^4 \). Then \( V^\perp \) contains only the vector **the zero vector**. Then \( (V^\perp)^\perp \) is \( V = \mathbb{R}^4 \).

3. Section 4.1, Problem 27

Answer: The lines \( 3x + y = b_1 \) and \( 6x + 2y = b_2 \) are **parallel**. They are the same line if \( 2b_1 = b_2 \). In that case \( (b_1, b_2) \) is perpendicular to the vector \( (2, -1) \). The nullspace of the matrix is the line \( 3x + y = 0 \). One particular vector in that nullspace is \( (-1, 3) \).

4. Section 4.2, Problem 14

Answer: The projection of \( b \) onto the column space of \( A \) is \( b \) itself, but \( P \) is not necessarily \( I \).

\[
P = \frac{1}{\Pi} \begin{bmatrix} 5 & 8 & -4 \\ 8 & 17 & 2 \\ -4 & 2 & 20 \end{bmatrix}
\]

and \( p = (0, 2, 4) \).
5. Section 4.2, Problem 25
Answer: The column space of \( P \) will be \( S \) (\( n \)-dimensional). Then \( r = \) dimension of the column space= \( n \).

6. Section 4.3, Problem 19
Answer: \( \hat{x} = (0, 0) \). If \( b = e \), then \( b \) is perpendicular to the column space of \( A \). The projection \( p = 0 \).

7. Section 4.3, Problem 20
Answer: \( \hat{x} = (9, 4) \) and \( e = (0, 0) \). The error \( e = (0, 0) \) because \( b \) is in the column space of \( A \).

8. Section 4.4, Problem 4
Answer:
\[
Q = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
0 & 0
\end{bmatrix},
QQ^T = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]
a) \( Q = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
0 & 0
\end{bmatrix},
QQ^T = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]
b) \( (1, 0) \) and \( (0, 0) \) are orthogonal but not independent.
c) \( \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right), \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right), \left( \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right), \left( -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) \)

9. Section 4.4, Problem 5
Answer: Two orthogonal vectors are \( (1, -1, 0) \) and \( (1, 1, -1) \). Orthonormal vectors are \( \left( \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 \right) \) and \( \left( \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right) \).

10. Section 5.1, Problem 2
Answer: If \( A \) is \( 3 \times 3 \) and \( \det(A) = -1 \), then
\[
\begin{align*}
\det(\frac{1}{2}A) &= (\frac{1}{2})^3\det(A) = -\frac{1}{8}, \\
\det(-A) &= (-1)^3\det(A) = -1, \\
\det(A^2) &= \det(A)\cdot\det(A) = 1, \\
\det(A^{-1}) &= 1/\det(A) = -1.
\end{align*}
\]

11. Section 5.1, Problem 29

Answer: If \( A \) is rectangular (not square), then \( \det(A), \det(A^T) \) are not defined.

12. Section 5.3, Problem 15

Answer:

a) Cofactors \( C_{21}, C_{31}, C_{32} \) are all zero.

b) \( C_{12} = C_{21}, C_{31} = C_{13}, C_{32} = C_{23} \).