1. (30 points)

(a) Find the matrix $P$ that projects every vector $b$ in $R^3$ onto the line in the direction of
$a = (2, 1, 3)$.

(b) What are the column space and nullspace of $P$? Describe them geometrically and also
give a basis for each space.

(c) What are all the eigenvectors of $P$ and their corresponding eigenvalues? (You can use
the geometry of projections, not a messy calculation.) The diagonal entries of $P$ add up
to ______.
2. (30 points)

(a) $p = A\hat{x}$ is the vector in $C(A)$ nearest to a given vector $b$. If $A$ has independent columns, what equation determines $\hat{x}$? What are all the vectors perpendicular to the error $e = b - A\hat{x}$? What goes wrong if the columns of $A$ are dependent?

(b) Suppose $A = QR$ where $Q$ has orthonormal columns and $R$ is upper triangular invertible. Find $\hat{x}$ and $p$ in terms of $Q$ and $R$ and $b$ (not $A$).

(c) (Separate question) If $q_1$ and $q_2$ are any orthonormal vectors in $R^5$, give a formula for the projection $p$ of any vector $b$ onto the plane spanned by $q_1$ and $q_2$ (write $p$ as a combination of $q_1$ and $q_2$).
3. (40 points) This problem is about the $n$ by $n$ matrix $A_n$ that has zeros on its main diagonal and all other entries equal to $-1$. In MATLAB $A_n = \text{eye}(n) - \text{ones}(n)$.

(a) Find the determinant of $A_n$. Here is a suggested approach:

Start by adding all rows (except the last) to the last row, and then factoring out a constant. (You could check $n = 3$ to have a start on part b.)

(b) For any invertible matrix $A$, the (1, 1) entry of $A^{-1}$ is the ratio of ________________ .

So the (1, 1) entry of $A^{-1}$ is ________________ .

(c) Find two orthogonal eigenvectors with $A_3 x = x$. (So $\lambda = 1$ is a double eigenvalue.)

(d) What is the third eigenvalue of $A_3$ and a corresponding eigenvector?