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(a) Find the projection $p$ of the vector $b$ onto the plane of $a_1$ and $a_2$, when

\[
b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad a_1 = \begin{bmatrix} 1 \\ 7 \\ 1 \\ 7 \end{bmatrix}, \quad a_2 = \begin{bmatrix} -1 \\ 7 \\ 1 \\ -7 \end{bmatrix}.
\]

(b) What projection matrix $P$ will produce the projection $p = Pb$ for every vector $b$ in $\mathbb{R}^4$?
(c) What is the determinant of $I - P$? Explain your answer.

(d) What are all nonzero eigenvectors of $P$ with eigenvalue $\lambda = 1$?

How is the number of independent eigenvectors with $\lambda = 0$ of an $n \times n$ square matrix $A$ connected to the rank of $A$?

(You could answer (c) and (d) even if you don’t answer (b).)
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2 (30 pts.)

(a) Suppose the matrix $A$ factors into $A = PLU$ with a permutation matrix $P$, and 1’s on the diagonal of $L$ (lower triangular) and pivots $d_1, \ldots, d_n$ on the diagonal of $U$ (upper triangular).

What is the determinant of $A$? EXPLAIN WHAT RULES YOU ARE USING.

(b) Suppose the first row of a new matrix $A$ consists of the numbers $1, 2, 3, 4$. Suppose the cofactors $C_{ij}$ of that first row are the numbers $2, 2, 2, 2$.

(Cofactors already include the ± signs.)

Which entries of $A^{-1}$ does this tell you and what are those entries?
(c) What is the determinant of the matrix $M(x)$? For which values of $x$ is the determinant equal to zero?

$$M(x) = \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & -1 & 2 & x \\
1 & 1 & 4 & x^2 \\
1 & -1 & 8 & x^3
\end{bmatrix}.$$
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3 (30 pts.)

(a) Starting from independent vectors \( a_1 \) and \( a_2 \), use Gram-Schmidt to find formulas for two orthonormal vectors \( q_1 \) and \( q_2 \) (combinations of \( a_1 \) and \( a_2 \)):

\[ q_1 = \]

\[ q_2 = \]

(b) The connection between the matrices \( A = [a_1 \ a_2] \) and \( Q = [q_1 \ q_2] \) is often written \( A = QR \). From your answer to Part (a), what are the entries in this matrix \( R \)?
(c) The least squares solution $\hat{x}$ to the equation $Ax = b$ comes from solving what equation?

If $A = QR$ as above, show that $R \hat{x} = Q^T b$. 
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