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(1) (40 pts)

(a) If $P$ projects every vector $b$ in $\mathbb{R}^5$ to the nearest point in the subspace spanned by $a_1 = (1, 0, 1, 0, 4)$ and $a_2 = (2, 0, 0, 0, 4)$, what is the rank of $P$ and why?

(b) If these two vectors are the columns of the 5 by 2 matrix $A$, which of the four fundamental subspaces for $A$ is the nullspace of $P$?

(c) By Gram-Schmidt find an orthonormal basis for the column space of $A$ (spanned by $a_1$ and $a_2$).

(d) If $P$ is any (symmetric) projection matrix, show that $Q = I - 2P$ is an orthogonal matrix.
(2) (30 pts.)

(a) Find the determinant of the matrix $A$

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & 2 & 3 & 0 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \end{bmatrix}.$$ 

(b) The absolute value of $\det A$ tells you the volume of a box in $\mathbb{R}^4$. Describe that box (2 points – describe a different box with the same volume).

(c) Suppose you remove row 3 and column 4 of an invertible 5 by 5 matrix $A$. If that reduced matrix is not invertible, what fact does that tell you about $A^{-1}$?
This 4 by 4 Hadmard matrix is an orthogonal matrix. Its columns are orthogonal unit vectors.

\[
Q = \frac{1}{2} \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 \\
1 & 1 & -1 & 1 \\
1 & -1 & -1 & 1
\end{bmatrix} = \begin{bmatrix}
q_1 \\
q_2 \\
q_3 \\
q_4
\end{bmatrix}
\]

(a) What projection matrix \( P_4 \) (give numbers) will project every \( b \) in \( \mathbb{R}^4 \) onto the line through \( q_4 \)?

(b) What projection matrix \( P_{123} \) will project every \( b \) in \( \mathbb{R}^4 \) onto the subspace spanned by \( q_1, q_2, \) and \( q_3 \)? Remember that those columns are orthogonal.

(c) Suppose \( A \) is the 4 by 3 matrix whose columns are \( q_1, q_2, q_3 \). Find the least-squares solution \( \hat{x} \) to the four equations

\[
Ax = \frac{1}{2} \begin{bmatrix}
1 & 1 & 1 \\
1 & -1 & 1 \\
1 & 1 & -1 \\
1 & -1 & -1
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix}
1 \\
2 \\
3 \\
4
\end{bmatrix} = b.
\]

What is the error vector \( e \)?