18.06 Spring 2013 – Problem Set 1

This problem set is due Thursday, February 14th, 2013 at 4pm (hand in to Room 2-255). The textbook problems are out of the 4th edition. Problems 1-8 are worth 8 points. Problems 9 and 10 are worth 18 points each.

1. Do Problem 32 from Section 2.2.
2. Do Problem 3 & Problem 7 from Section 2.3.
3. Do Problem 11 from Section 2.4.
4. Do Problem 33 from Section 2.4.
5. Do Problem 36 from Section 2.4.
6. Do Problem 8 from Section 2.5.
7. Do Problem 25 from Section 2.5.
8. Do Problem 40 from Section 2.5.
9. The $3 \times 3$ matrix $A$ is given as the sum of two other $3 \times 3$ matrices $B$ and $C$ satisfying:
   - all rows of $B$ are the same vector $u$
   - all columns of $C$ are the same vector $v$.

   Show that $A$ is not invertible. One possible approach is to explain why there is a nonzero vector $x$ satisfying both $Bx = 0$ and $Cx = 0$, so that $Ax = (B + C)x = Bx + Cx = 0$ has a nonzero solution.

10. A matrix $A$ is called symmetric when its rows are the same as its columns. If we denote the entry in the $i$-th row and $j$-th column in $A$ as $a_{ij}$, this means that $a_{ij} = a_{ji}$. For example,

   \[
   \begin{bmatrix}
   1 & 4 & 5 & 8 \\
   4 & 2 & 3 & 6 \\
   5 & 3 & 7 & 2013 \\
   8 & 6 & 2013 & 0
   \end{bmatrix}
   \]

   is a symmetric matrix (here, $a_{34} = a_{43} = 2013$).

   $A$ is tridiagonal when all the entries of $A$ except in the middle three diagonals are zero. This means that $a_{ij} = 0$ if $|i - j| > 1$. An example of a tridiagonal matrix is

   \[
   \begin{bmatrix}
   1 & 4 & 0 & 0 & 0 \\
   6 & 2 & 7 & 0 & 0 \\
   0 & \pi & 7 & 2013 & 0 \\
   0 & 0 & 7.5 & 4 & 9 \\
   0 & 0 & 0 & 11 & 15
   \end{bmatrix}
   \]

   **Problem.** Construct a $3 \times 3$ tridiagonal matrix $A$ with pivots 3, 4, and 5 so that performing elimination steps on $A$ goes:
• subtract row 1 from row 2.
• subtract row 2 from row 3.