18.06 (Spring 2014) Problem Set 2
These 8 problems are worth 80 points. MITx problems are worth 20 points. This problem set is due on Thursday, Feb 20, 2014 by 4pm in E17-131.

1. Imagine that the 2nd difference matrix $S$ (with 1, $-2, 1$ down three central diagonals) is INFINITE. Multiply $S$ with these infinite vectors (infinite in both directions):

(a) all-ones ($..., 1, 1, 1, 1, ...$)
(b) linear ($..., -2, -1, 0, 1, 2, 3, ...$)
(c) squares ($..., 4, 1, 0, 1, 4, 9, ...$)
(d) cubes ($..., -8, -1, 0, 1, 8, 27, ...$)

How do the answers match up with 2nd derivatives of $1, x, x^2, x^3$?

2. Find the inverse of the 4 by 4 backward difference matrix $B$ (main diagonal of 1's and subdiagonal of $-1$'s). Interpret as the fundamental theorem of calculus. The inverse of the derivative is ____________.

3. If the permutation $P$ has 1's on the antidiagonal (from the $(1, n)$ entry down to the $(n, 1)$ entry) is this an even or odd permutation (depending on $n$)?

4. Find the pivots, multipliers, $LU$ factors, and determinant of $A$:

$$A = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix}.$$ 

5. Now let

$$A = \begin{pmatrix} 4 & 10 & 0 \\ 8 & b & 4 \\ 4 & 0 & 1 \end{pmatrix}.$$ 

What value of $b$ interferes with normal elimination? What should you do in this case? Which $b$ makes the matrix singular?

6. Problem 30 page 91 (Section 2.5)

7. Problem 40 page 92 (Section 2.5)

8. 

(a) Suppose every row of $A$ adds up to zero. Why is $A$ singular?

(b) Suppose every column of $A$ adds to zero. Why is $A$ singular?

9. MATLAB problems (20 pts): please go to lms.mitx.mit.edu to complete the problems.