18.06 Exam IV: Eigenstuff
29 April 2016

STATE YOUR NAME: ________________________________

R01 10–11 Sauer–Ayala
R02 10–11 Carpenter
R03 11–12 Sauer–Ayala
R04 11–12 Carpenter
R05 12–13 Hopkins
R06 12–13 Anno
R07 13–14 Hopkins
R08 13–14 Anno
R09 14–15 Fei
R10 14–15 Knizel
R11 15–16 Knizel

CIRCLE YOUR RECITATION:

GRADING

1. _______ /20
2. _______ /20
3. _______ /20
4. _______ /20
5. _______ /20

TOTAL

/100
1. Yah or neh?

For each of the following sentences, indicate whether they are true or false. (No need to justify your answer.)

(a) If $A$ is an $n \times n$ matrix in which one row is a multiple of another, then $\det(A) = 0$.

(b) If $A$ is a real $n \times n$ matrix, then $\det(A^2) \geq 0$.

(c) If $A$ is an $n \times n$ matrix with $n$ distinct real eigenvalues, then $A$ is diagonalizable over $\mathbb{R}$.

(d) There are matrices that are diagonalizable over $\mathbb{C}$ but not diagonalizable over $\mathbb{R}$.

(e) The characteristic polynomial of an $n \times n$ matrix with complex entries has degree $n$. 

2. **Determine ant**

Compute

\[
\begin{vmatrix}
0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1
\end{vmatrix}
\]
3. **Dudley Eigenvalue, DDS**

Compute the eigenvalues and eigenspaces of

\[
\begin{pmatrix}
1 & 9 \\
4 & 1 \\
\end{pmatrix}
\quad \text{and} \quad
\begin{pmatrix}
1 & 9 & 0 \\
4 & 1 & 0 \\
0 & 0 & -2
\end{pmatrix}.
\]

Are these matrices diagonalizable over \( \mathbb{R} \)? Over \( \mathbb{C} \)?
4. Complications

Compute the eigenvalues and eigenspaces of

\[
\begin{pmatrix}
3 & -2 & 0 & 0 \\
2 & 3 & 0 & 0 \\
0 & 0 & -2 & -1 \\
0 & 0 & 1 & -2
\end{pmatrix}.
\]

Is this matrix diagonalizable over \( \mathbb{R} \)? Over \( \mathbb{C} \)?
5. Vade mecum

Compute the eigenvalues and eigenspaces of

\[
\begin{pmatrix}
0 & 0 & 0 & -1 \\
1 & 0 & 0 & -4 \\
0 & 1 & 0 & -6 \\
0 & 0 & 1 & -4
\end{pmatrix}.
\]