Your name is: ____________________________ Grading 1

Please circle your recitation:

1) Mon 2–3 2-131 S. Kleiman 5) Tues 12–1 2-131 S. Kleiman
2) Mon 3–4 2-131 S. Hollander 6) Tues 1–2 2-131 S. Kleiman
3) Tues 11–12 2-132 S. Howson 7) Tues 2–3 2-132 S. Howson
4) Tues 12–1 2-132 S. Howson

1 (30 pts.) (a) Compute the determinant of

\[
A = \begin{bmatrix}
1 & -1 & 1 & 0 \\
1 & 1 & 5 & 0 \\
1 & 3 & 9 & 0 \\
0 & 0 & 0 & 2 \\
\end{bmatrix}.
\]

(b) Find an orthogonal basis (orthonormal is even better) for the column space of \(A\). Start from a basis and use Gram-Schmidt (and common sense).

(c) If you change the 1 in the upper left corner of \(A\) to 2, what is the change in the determinant (I would use cofactors).
2 (24 pts.) An experiment at the nine times $t = -4, -3, -2, -1, 0, 1, 2, 3, 4$ yields the consistent result $b = 0$ except at the last time ($t = 4$) we get $b = 10$. We want the best straight line $b = C + Dt$ to fit these nine data points by least squares.

(a) Write down the equations $Ax = b$ with unknowns $C$ and $D$ that would be solved if a straight line exactly fit the data (it doesn’t).

(b) Find the best least squares value of $C$ and $D$.

(c) This problem is really projecting the vector $b = (0, 0, 0, 0, 0, 0, 0, 0, 10)$ onto a certain subspace. Give a basis for that subspace and give the projection $p$ of $b$ onto the subspace.
3 (22 pts.) Suppose an \( m \) by \( n \) matrix \( Q \) has orthonormal columns.

(a) What is the rank of \( Q \)?

(b) Give an expression with no inverses for the projection matrix \( P \) onto the column space of \( Q \).

(c) Check that your formula for \( P \) satisfies the two requirements for a projection matrix.
(a) Suppose $Q$ is an orthogonal matrix and $Qx = \lambda x$. Compare the lengths of $\lambda x$ and $Qx$ (using $(Qx)^T(Qx)$) to reach a conclusion about $\lambda$.

(b) The Hadamard matrix $H$ has orthogonal columns:

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}.$$ 

Project the vector $b = (1, 2, 3, 4)$ onto the line spanned by the last column. Then project $b$ onto the subspace spanned by all four columns.

(c) Find the eigenvalues of $H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.