Problem 1

This problem is aimed at helping you picture the different ways in which controls fit into everyday life. This answers are not exhaustive of the possibilities. In particular for the robot welder, there are many other possibilities on what to control in addition to position, such as strength of weld, duration, speed, heat used.

<table>
<thead>
<tr>
<th></th>
<th>Water closet</th>
<th>Refrigerator</th>
<th>Shower mixer</th>
<th>Robot Welder</th>
<th>Iris of your eye</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Engineering/customer goal</td>
<td>To flush toilet each time with appropriate volume, refill tank</td>
<td>Keep food cold so it won’t spoil</td>
<td>Mixing of hot and cold water to maintain desired temp. and volume</td>
<td>Make the weld in the correct position each time</td>
</tr>
<tr>
<td>b</td>
<td>A measure of good performance</td>
<td>Height of water in tank after each flush</td>
<td>Maintain temperature inside relatively constant</td>
<td>Proper amount of flow at a constant temp.</td>
<td>Position of weld from desired position</td>
</tr>
<tr>
<td>c</td>
<td>Output variable</td>
<td>Water height</td>
<td>Temperature inside box</td>
<td>Volume rate and temperature of water</td>
<td>Position of weld</td>
</tr>
<tr>
<td>d</td>
<td>Reference input</td>
<td>Water height set by floater</td>
<td>Thermostat</td>
<td>Position of shower knob. (this input is usually not in degrees, but rather relative position)</td>
<td>Position of where weld should be as programmed by the user</td>
</tr>
<tr>
<td>e</td>
<td>Plant and actuator</td>
<td>Water tank and valve</td>
<td>Icebox and cooling system</td>
<td>Shower head, knob, valve</td>
<td>Robot and piece to weld</td>
</tr>
<tr>
<td>f</td>
<td>Measurement</td>
<td>Floater</td>
<td>Thermometer</td>
<td>The user feels the correct water temp, volume rate</td>
<td>Some type of position sensor</td>
</tr>
<tr>
<td>g</td>
<td>Disturbance</td>
<td>Floater gets stuck</td>
<td>Opening the door</td>
<td>Loss of hot water in water input, Drop in water pressure</td>
<td>Vibration</td>
</tr>
</tbody>
</table>
Problem 2

Functional Block Diagram:

Input Transducer: Pilot will input a desired angle through his controls which will be converted to an input voltage.
Output Transducer: Gyroscope measures the actual angle and converts it to a voltage
Controller: Aileron position
Plant: Plane dynamics

Problem 3

Functional Block Diagram

It is important to understand and appreciate the difference between functional block diagrams and mathematical block diagrams. The latter will be the focus of this course, but Functional block diagrams are particularly useful in the early design process of your control systems.
Problem 4

a) Time Constant
Note that this is a **first order** system which can be rewritten to fit the general form of:
\[
\frac{d}{dt} X + \frac{1}{\tau} X = \text{input}
\]

The equation becomes:
\[
\frac{d}{dt} T + \frac{1}{60} T = \frac{1}{18} Q_i
\]

Then it is obvious that \( \tau = 60 \).

b) Steady State Gain (K)
You can find the steady state gain by letting all the transients go to zero. Namely \( \frac{d}{dt} = 0 \).

Therefore:
\[
K = \frac{T}{Q} = \frac{10}{3}
\]

c) Transfer Function
Find the transfer function by applying the Laplace transform:
\[
sT(s) + \frac{1}{60} T(s) = \frac{1}{18} Q(s)
\]

Then solving for the transfer function
\[
G(s) = \frac{T(s)}{Q(s)} = \frac{1/18}{s + 1/60}.
\]

d) Plot
The equation for the curve is
\[
T(t) = \frac{10}{3} - \frac{10}{3} e^{-\frac{t}{60}}
\]

<table>
<thead>
<tr>
<th>Value of t/( \tau )</th>
<th>t</th>
<th>Value of function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
<td>2.0865</td>
</tr>
<tr>
<td>2</td>
<td>120</td>
<td>2.8746</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
<td>3.1646</td>
</tr>
<tr>
<td>4</td>
<td>240</td>
<td>3.2713</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
<td>3.3105</td>
</tr>
</tbody>
</table>
Problem 5
Note that this is a second order system.
a) Equation of motion

Equation of motion is found by doing a force balance on the system

\[ F_m = F_x + F_k \]
\[ m\ddot{x}_2 = k(x_1 - x_2) + b(\dot{x}_1 - \dot{x}_2) \]

which is the same as:
\[ m\ddot{v}_2 = k(\int v_1 - \int v_2) + b(v_1 - v_2) \]

b) Transfer function

The transfer function can be obtained by taking the Laplace transform

\[ G(s) = \frac{Y(s)}{U(s)} = \frac{V_2(s)}{V_1(s)} = \frac{k + bs}{ms^2 + bs + k} \]

c) System Parameters
Given the system parameters: \( k = 400 \text{ N/m}, m = 1 \text{ Kg}, b = 300 \text{ N/m/sec} \)

We can calculate all the relevant information about the system.
First we change the denominator so that it looks like the general second order form, namely:

\[ s^2 + 2\zeta\omega_n s + \omega_n^2 \]

\[ G(s) = \frac{V_2(s)}{V_1(s)} = \frac{k/m + b/m s}{s^2 + (b/m)s + (k/m)} \]

Then we can easily figure out what each term is:

\[ G(s) = \frac{V_2(s)}{V_1(s)} = \frac{300s + 400}{s^2 + 300s + 400} \]

\[ \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{400 \text{ N/m}}{1 \text{ Kg}}} = 20 \text{ Hz} \]

\[ \zeta = \frac{1}{2\omega_n} \frac{b}{m} = \frac{1}{40 \text{ Hz}} \frac{300 \text{ N/m/sec}}{1 \text{ Kg}} = 7.5 \text{ (VERY overdamped)} \]

Approximation for the 2% settling time for a second order system is given by:

\[ T_s = \frac{4}{\zeta\omega_n} = \frac{4}{7.5 \times 20} = 0.0267 \]
d) Step Response

\[ T(t) = L^{-1}\left\{ \frac{300s + 400}{s^2 + 300s + 400} \frac{1}{s} \right\} \]

f) Bode Plot
Notice that since the system is highly overdamped there is no peak for this second order system.
g) Resonant Frequency
The resonant frequency of a second order system is at the peak of the magnitude of the bode plot. Since the system is overdamped it will not resonate.